Series **ABCD1/2**

Set No. 3

प्रश्न-पत्र कोड
Q.P. Code **65/2/3**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **7** हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **14** प्रश्न हैं ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **7** printed pages.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- *Please check that this question paper contains **14** questions.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 2 घण्टे | *अधिकतम अंक : 40*

Time allowed : 2 hours | *Maximum Marks : 40*

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र के **तीन** खण्ड हैं – **खण्ड क, ख** तथा **ग** ।*

(ii) *प्रत्येक खण्ड **अनिवार्य** है ।*

(iii) ***खण्ड क** में **6** लघु उत्तर I प्रकार के प्रश्न हैं जिनमें प्रत्येक के **2** अंक हैं ।*

(iv) ***खण्ड ख** में **4** लघु उत्तर II प्रकार के प्रश्न हैं जिनमें प्रत्येक के **3** अंक हैं ।*

(v) ***खण्ड ग** में **4** दीर्घ-उत्तरीय प्रकार के प्रश्न हैं जिनमें प्रत्येक के **4** अंक हैं ।*

(vi) *कुछ प्रश्नों में आंतरिक विकल्प दिया गया है ।*

(vii) *प्रश्न सं. **14** एक प्रकरण-अध्ययन आधारित प्रश्न है जिसमें दो भाग हैं । प्रत्येक भाग के **2** अंक हैं ।*

खण्ड क

*प्रश्न संख्या **1** से **6** तक प्रत्येक प्रश्न के **2** अंक हैं ।*

1. रेखा PQ जो बिन्दुओं P(2, 2, 1) तथा Q(5, 1, – 2) से गुज़रती है, का कार्तीय समीकरण लिखिए । अतः रेखा PQ पर उस बिन्दु का y-निर्देशांक ज्ञात कीजिए जिसका z-निर्देशांक – 2 है । *2*

2. एक विशेष प्रश्न को A और B द्वारा स्वतंत्र रूप से हल करने की प्रायिकताएँ क्रमशः $\frac{2}{3}$ और $\frac{3}{5}$ हैं । यदि दोनों स्वतंत्र रूप से प्रश्न को हल करने की कोशिश करते हैं, तो प्रश्न के हल हो जाने की प्रायिकता ज्ञात कीजिए । *2*

3. पासों के एक युग्म को उछाला गया । दोनों पासों पर आई संख्याओं का योगफल एक सम संख्या है । कम-से-कम एक पासे पर संख्या 3 आने की प्रायिकता ज्ञात कीजिए । *2*

4. दो मात्रक सदिशों $\vec{a}$ और $\vec{b}$ के लिए $|2\vec{a} + 3\vec{b}| = |3\vec{a} - 2\vec{b}|$ है । $\vec{a}$ तथा $\vec{b}$ के बीच का कोण ज्ञात कीजिए । *2*

5. अवकल समीकरण

$$\left[\frac{d}{dx}(xy^2)\right] \cdot \frac{dy}{dx} + y = 0$$

की कोटि और घात का गुणनफल ज्ञात कीजिए । *2*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* ***three*** *sections –* ***Section A, B*** *and* ***C****.*

(ii) *Each section is* ***compulsory****.*

(iii) ***Section A*** *has* ***6*** *short answer type I questions of* ***2*** *marks each.*

(iv) ***Section B*** *has* ***4*** *short answer type II questions of* ***3*** *marks each.*

(v) ***Section C*** *has* ***4*** *long answer type questions of* ***4*** *marks each.*

(vi) *There is an internal choice in some questions.*

(vii) *Question no.* ***14*** *is a case-study based question with 2 sub-parts of* ***2*** *marks each.*

SECTION A

Questions number ***1*** *to* ***6*** *carry* ***2*** *marks each.*

1. Write the cartesian equation of the line PQ passing through points P(2, 2, 1) and Q(5, 1, – 2). Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is -2. *2*

2. Probabilities of A and B solving a specific problem are $\frac{2}{3}$ and $\frac{3}{5}$, respectively. If both of them try independently to solve the problem, then find the probability that the problem is solved. *2*

3. A pair of dice is thrown. It is given that the sum of numbers appearing on both dice is an even number. Find the probability that the number appearing on at least one die is 3. *2*

4. $\vec{a}$ and $\vec{b}$ are two unit vectors such that $|2\vec{a} + 3\vec{b}| = |3\vec{a} - 2\vec{b}|$. Find the angle between $\vec{a}$ and $\vec{b}$. *2*

5. Find the product of the order and the degree of the differential equation $\left[\frac{d}{dx}(xy^2)\right] \cdot \frac{dy}{dx} + y = 0$. *2*

6. (क) ज्ञात कीजिए : *2*

$$\int \frac{\sqrt{\cot x}}{\sin x \cos x} dx$$

अथवा

(ख) ज्ञात कीजिए : *2*

$$\int \frac{1}{x(x^2+4)} dx$$

खण्ड ख

*प्रश्न संख्या **7** से **10** तक प्रत्येक प्रश्न के **3** अंक हैं ।*

7. (क) मान ज्ञात कीजिए : *3*

$$\int_0^1 \tan^{-1} x \, dx$$

अथवा

(ख) ज्ञात कीजिए : *3*

$$\int \frac{2x}{x^2+3x+2} dx$$

8. (क) बिन्दुओं (2, 1, 0), (3, – 2, – 2) और (1, 1, – 7) से गुज़रने वाले समतल का समीकरण ज्ञात कीजिए । इस समतल की मूल-बिन्दु से दूरी भी ज्ञात कीजिए । *3*

अथवा

(ख) रेखाओं $x = \frac{y-1}{2} = \frac{z-2}{3}$ तथा $x + 1 = \frac{y+2}{2} = \frac{z-1}{3}$ के बीच की दूरी ज्ञात कीजिए । *3*

9. निम्नलिखित अवकल समीकरण हल कीजिए : *3*

$$(1 + e^{y/x})\, dy + e^{y/x}\left(1 - \frac{y}{x}\right) dx = 0$$

10. यदि $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ और $\vec{b} = 2\hat{i} - \hat{j} - 3\hat{k}$ दो सदिश हैं, तो सदिश $\vec{c}$ ज्ञात कीजिए, जिसके लिए $\vec{a} \times \vec{c} = \vec{b}$ और $\vec{a} \cdot \vec{c} = 4$ दिया गया है । *3*

6. (a) Find : *2*

$$\int \frac{\sqrt{\cot x}}{\sin x \cos x}\, dx$$

OR

(b) Find : *2*

$$\int \frac{1}{x\,(x^2 + 4)}\, dx$$

SECTION B

*Questions number **7** to **10** carry **3** marks each.*

7. (a) Evaluate : *3*

$$\int_0^1 \tan^{-1} x\, dx$$

OR

(b) Find : *3*

$$\int \frac{2x}{x^2 + 3x + 2}\, dx$$

8. (a) Find the equation of the plane passing through points (2, 1, 0), (3, – 2, – 2) and (1, 1, – 7). Also, obtain its distance from the origin. *3*

OR

(b) Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x + 1 = \frac{y+2}{2} = \frac{z-1}{3}$. *3*

9. Solve the following differential equation : *3*

$$(1 + e^{y/x})\, dy + e^{y/x}\left(1 - \frac{y}{x}\right) dx = 0$$

10. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} - 3\hat{k}$, then find the vector $\vec{c}$, given that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 4$. *3*

खण्ड ग

*प्रश्न संख्या **11** से **14** तक प्रत्येक प्रश्न के **4** अंक हैं ।*

11. मान ज्ञात कीजिए : *4*

$$\int_0^1 x(1-x)^n \, dx$$

12. (क) समाकलन के प्रयोग से, वक्र $4x^2 + 4y^2 = 9$ और रेखा $2x + 2y = 3$ के बीच घिरे लघु क्षेत्र का क्षेत्रफल ज्ञात कीजिए । *4*

अथवा

(ख) यदि वक्र $y^2 = 4ax$ और रेखा $x = 4a$ से घिरे क्षेत्र का क्षेत्रफल $\frac{256}{3}$ वर्ग इकाई है, तो समाकलन के प्रयोग से a, जहाँ $a > 0$, का मान ज्ञात कीजिए । *4*

13. उस बिन्दु के निर्देशांक ज्ञात कीजिए जहाँ बिन्दुओं (5, 1, 6) तथा (3, 4, 1) से गुज़रती रेखा zx-समतल को काटती है । *4*

प्रकरण-अध्ययन आधारित प्रश्न

14. किसी भी क्रिकेट के मैच के आरम्भ में, एक सिक्का उछाला जाता है और जो टीम टॉस जीतती है उसे बल्लेबाजी या गेंदबाजी चुनने का मौका मिलता है । ऐसा सिक्का अनभिनत होता है जिसमें चित और पट के आने की प्रायिकताएँ समान होती हैं ।

उपरोक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(क) यदि ऐसा सिक्का 2 बार उछाला जाए, तो पटों की संख्या के लिए प्रायिकता बंटन ज्ञात कीजिए । *2*

(ख) ऐसे सिक्के को 3 बार उछालने पर कम-से-कम एक चित आने की प्रायिकता ज्ञात कीजिए । *2*

SECTION C

*Questions number **11** to **14** carry **4** marks each.*

11. Evaluate : *4*

$$\int_0^1 x(1-x)^n \, dx$$

12. (a) Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$. *4*

OR

(b) If the area of the region bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$. *4*

13. Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the zx-plane. *4*

Case-Study Based Question

14. At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. Such a coin is unbiased with equal probabilities of getting head and tail.

Based on the above information, answer the following questions :

(a) If such a coin is tossed 2 times, then find the probability distribution of number of tails. *2*

(b) Find the probability of getting at least one head in three tosses of such a coin. *2*

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Senior Secondary School Term II Examination, 2022

Marking Scheme – MATHEMATICS (SUBJECT CODE – 041)

(PAPER CODE – 65/2/3)

General Instructions: -

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.

2. **"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under IPC."**

3. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them. In class-XII, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, marks should be awarded.**

4. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.

5. Evaluators will mark (√) wherever answer is correct. For wrong answer 'X' be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**

6. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totalled up and written in the left-hand margin and encircled. This may be followed strictly.

7. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

8. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.

9. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

10. A full scale of marks _____(example 0-40 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.

11. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 30 answer books per day in main subjects and 35 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.

12. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-

    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totalling of marks awarded on a reply.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totalling on the title page.
    - Wrong totalling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.

13. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.

14. Any unassessed portion, non-carrying over of marks to the title page, or totalling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.

15. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.

16. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totalled and written in figures and words.

17. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

MARKING SCHEME

Senior Secondary School Examination TERM–II, 2022

MATHEMATICS (Subject Code–041)

Paper Code: 65/2/3]

Maximum Marks: 40

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION – A	
1.	Write the cartesian equation of the line PQ passing through points P(2, 2, 1) and Q(5, 1, – 2). Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is – 2.	2
Sol.	Required equation of line is given by $$\frac{x-2}{3}=\frac{y-2}{-1}=\frac{z-1}{-3}$$ putting $z=-2$, we get $\frac{y-2}{-1}=\frac{-3}{-3}=1$ $y-2=-1 \Rightarrow y=1$	1 ½ ½ 2
2.	Probabilities of A and B solving a specific problem are $\frac{2}{3}$ and $\frac{3}{5}$, respectively. If both of them try independently to solve the problem, then find the probability that the problem is solved.	2
Sol.	P (Problem is solved) $=1-P$ (Problem not solved) $=1-P(\bar{A})P(\bar{B})$ $=1-\frac{1}{3}\cdot\frac{2}{5}$ $=\frac{13}{15}$	1 1 2
3.	A pair of dice is thrown. It is given that the sum of numbers appearing on both dice is an even number. Find the probability that the number appearing on at least one die is 3.	2
Sol.	A: number appearing on at least one dice is 3 B: sum of numbers appearing on both dice is even Clearly, $A \cap B = \{(3,1),(3,5),(1,3),(5,3),(3,3)\}$	½

	$P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)} = \frac{5/36}{18/36}$ $= \frac{5}{18}$	1 ½ 2
4.	$\vec{a}$ and $\vec{b}$ are two unit vectors such that $\lvert 2\vec{a} + 3\vec{b} \rvert = \lvert 3\vec{a} - 2\vec{b} \rvert$. Find the angle between $\vec{a}$ and $\vec{b}$.	2
Sol.	$\lvert 2\vec{a} + 3\vec{b}\rvert = \lvert 3\vec{a} - 2\vec{b}\rvert$ $\Rightarrow \lvert 2\vec{a} + 3\vec{b}\rvert^2 = \lvert 3\vec{a} - 2\vec{b}\rvert^2$ $\Rightarrow 4\lvert\vec{a}\rvert^2 + 12\,\vec{a}.\vec{b} + 9\lvert\vec{b}\rvert^2 = 9\lvert\vec{a}\rvert^2 - 12\,\vec{a}.\vec{b} + 4\lvert\vec{b}\rvert^2$ As $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = 1$ $\therefore 24\,\vec{a}.\vec{b} = 5\lvert\vec{a}\rvert^2 - 5\lvert\vec{b}\rvert^2 = 0 \Rightarrow \vec{a}.\vec{b} = 0$ So, $\vec{a} \perp \vec{b}$ or Angle between them is $\frac{\pi}{2}$	1 ½ ½ 2
5.	Find the product of the order and the degree of the differential equation $\left[\frac{d}{dx}(xy^2)\right].\frac{dy}{dx} + y = 0$.	2
Sol.	Given differential equation can be written as $2xy\left(\frac{dy}{dx}\right)^2 + y^2\frac{dy}{dx} + y = 0$ Order = 1, Degree = 2 Order × degree = 1 × 2 = 2	1 1 2
6.(a)	Find : $\int \frac{\sqrt{\cot x}}{\sin x \cos x}\,dx$	2
Sol.	Let $\cot x = t$, then $-\operatorname{cosec}^2 x\,dx = dt$ $\therefore \int \frac{\sqrt{\cot x}}{\sin x \cos x}\,dx = \int \frac{\sqrt{\cot x}}{\cot x}\operatorname{cosec}^2 x\,dx = -\int \frac{\sqrt{t}}{t}\,dt = -\int \frac{1}{\sqrt{t}}\,dt$ $= -2\sqrt{t} + C$ $= -2\sqrt{\cot x} + C$	½ ½ ½ ½ 2
	Or	

6.(b)	Find : $$\int \frac{1}{x\,(x^2+4)}\,dx$$	2
Sol.	Let $I = \int \frac{1}{x(x^2+4)}dx = \int \frac{1}{x^3\left(1+\frac{4}{x^2}\right)}dx$ Let $1+\frac{4}{x^2} = t \Rightarrow \frac{-8}{x^3}dx = dt$ $\therefore\ I = \frac{-1}{8}\int \frac{dt}{t}$ $= \frac{-1}{8}\log\lvert t\rvert + C$ $= \frac{-1}{8} log\left[1+\frac{4}{x^2}\right] + C$	½ ½ ½ ½
		2
	SECTION – B	
7.(a)	Evaluate : $$\int_0^1 \tan^{-1} x\,dx$$	3
Sol.	Consider $\int(\tan^{-1}x)\,dx = \tan^{-1}x\cdot x - \int \frac{1}{1+x^2}\cdot x\,dx$ $= x\,tan^{-1}x - \frac{1}{2}\log(1+x^2)$ $\int_0^1(\tan^{-1}x)\,dx = x\tan^{-1}x - \frac{1}{2}\log\lvert 1+x^2\rvert\Big]_0^1$ $= \frac{\pi}{4} - \frac{1}{2}\log 2$	1 1 1 3
	Or	

7.(b)	Find : $$\int \frac{2x}{x^2+3x+2}\,dx$$	3
Sol.	$I=\int \frac{2x\cdot dx}{x^2+3x+2}=\int \frac{2x}{(x+1)(x+2)}dx$ $=\int\left(\frac{-2}{x+1}+\frac{4}{x+2}\right)dx$ using partial fraction $=-2\log\lvert x+1\rvert+4\log\lvert x+2\rvert+C$	½ 1½ 1 3
8.(a)	Find the equation of the plane passing through points (2, 1, 0), (3, – 2, – 2) and (1, 1, – 7). Also, obtain its distance from the origin.	3
Sol.	Equation of plane is given by $\begin{vmatrix} x-2 & y-1 & z \\ 1 & -3 & -2 \\ -1 & 0 & -7 \end{vmatrix}=0$ $21(x-2)+9(y-1)-3z=0$ i.e., $7x+3y-z=17$ Distance of plane from origin is $d=\frac{\lvert 0+0-0-17\rvert}{\sqrt{59}}=\frac{17}{\sqrt{59}}$	1½ 1 ½ 3
	Or	
8.(b)	Find the distance between the lines $x=\frac{y-1}{2}=\frac{z-2}{3}$ and $x+1=\frac{y+2}{2}=\frac{z-1}{3}$.	3
Sol.	For lines $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and $\frac{x+1}{1}=\frac{y+2}{2}=\frac{z-1}{3}$ Let $\vec{a_1}=\hat{j}+2\hat{k},\vec{b}=\hat{\imath}+2\hat{j}+3\hat{k}$ $\vec{a_2}=-\hat{\imath}-2\hat{j}+\hat{k},\vec{b}=\hat{\imath}+2\hat{j}+3\hat{k}$ Clearly lines are parallel Hence, Shortest distance or distance is given by $\frac{\lvert(\vec{a_2}-\vec{a_1})\times\vec{b}\rvert}{\lvert\vec{b}\rvert}$	½

	$\vec{a_2} - \vec{a_1} = -\hat{\imath} - 3\hat{\jmath} - \hat{k}$ $(\vec{a_2} - \vec{a_1}) \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & -3 & -1 \\ 1 & 2 & 3 \end{vmatrix}$ $= -7\hat{\imath} + 2\hat{\jmath} + \hat{k}$ Required distance = $\frac{\sqrt{49+4+1}}{\sqrt{1+4+9}} = \frac{\sqrt{27}}{\sqrt{7}}$ or $\frac{3\sqrt{21}}{7}$	½ 1 1 3
9.	Solve the following differential equation : $(1 + e^{y/x})\, dy + e^{y/x}\left(1 - \frac{y}{x}\right) dx = 0$	3
Sol.	Given differential equation can be written as $\frac{dy}{dx} = \frac{e^{y/x}\left(\frac{y}{x} - 1\right)}{1 + e^{y/x}}$ put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get $v + x\frac{dv}{dx} = \frac{e^v(v-1)}{1+e^v}$ $x\frac{dv}{dx} = \frac{-e^v - v}{1+e^v}$ $\int \frac{(e^v+1)}{e^v+v} dv = -\int \frac{dx}{x}$ $\log\lvert e^v + v\rvert = -\log\lvert x\rvert + \log C$ $e^v + v = \frac{C}{x}$ $e^{y/x} + \frac{y}{x} = \frac{C}{x}$ or $xe^{y/x} + y = C$	½ ½ ½ 1 ½ 3
10.	If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} - 3\hat{k}$, then find the vector $\vec{c}$, given that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 4$.	3
Sol.	Let $\vec{c} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$	½ ½

	$\vec{a}\cdot\vec{c} = 4 \Rightarrow x - y + z = 4$ $\vec{a}\times\vec{c} = (-z-y)\hat{\imath} + (x-z)\hat{\jmath} + (x+y)\hat{k}$ $\vec{a}\times\vec{c} = \vec{b} \Rightarrow \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 1 \\ x & y & z \end{vmatrix} = (2\vec{\imath} - \hat{\jmath} - 3\hat{k})$ $\Rightarrow -(y+z)\hat{\imath} - (z-x)\hat{\jmath} + (y+x)\hat{k} = 2\vec{\imath} - \hat{\jmath} - 3\hat{k}$ $\Rightarrow y+z = -2, z-x = 1, y+x = -3$ Solving we get, $x = 0,\ y = -3,\ z = 1$ $\therefore \vec{c} = -3\hat{\jmath} + \hat{k}$	1 ½ ½ 3
	SECTION – C	
11.	Evaluate : $\int_0^1 x(1-x)^n\,dx$	4
Sol.	$I = \int_0^1 x(1-x)^n dx$ $= \int_0^1 (1-x)[1-(1-x)]^n dx$ [using property] $= \int_0^1 x^n(1-x)\,dx$ $= \int_0^1 x^n dx - \int_0^1 x^{n+1} dx$ $= \left[\frac{x^{n+1}}{n+1}\right]_0^1 - \left[\frac{x^{n+2}}{n+2}\right]_0^1$ $= \frac{1}{n+1} - \frac{1}{n+2}$ Or $\frac{1}{(n+1)(n+2)}$	1 1 1 1 4
12.(a)	Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.	4
Sol.	Clearly point of intersection are $\left(\frac{3}{2}, 0\right)$ & $\left(0, \frac{3}{2}\right)$ 	½

	Y′ Correct Figure Required area = $\int_0^{3/2}\sqrt{\frac{9}{4}-x^2}\,dx-\int_0^{3/2}\left(\frac{3}{2}-x\right)dx$ $=\frac{x}{2}\sqrt{\frac{9}{4}-x^2}+\frac{9}{8}\sin^{-1}\frac{2x}{3}\Big\|_0^{3/2}+\frac{\left(\frac{3}{2}-x\right)^2}{2}\Big\|_0^{3/2}$ $=\frac{9\pi}{16}-\frac{9}{8}$	1 1 1 ½ 4
	Or	
12.(b)	If the area of the region bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$.	4
Sol.	Given area = $\frac{256}{3}$ Correct Figure Area of Shaded region = $2\int_0^{4a}\sqrt{4ax}\,dx$ $=8\sqrt{a}\,\frac{x^{3/2}}{3}\Big\|_0^{4a}$ $=\frac{64a^2}{3}$	1 1 1 ½

	$\frac{64a^2}{3} = \frac{256}{3}$ $\Rightarrow a^2 = 4$ gives $a = 2$ (as $a > 0$)	½ 4
13.	Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the zx-plane.	4
Sol.	Equations of line $\frac{x-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5}$ Any point on the line $(-2\lambda + 5, 3\lambda + 1, -5\lambda + 6)$ The point lies on ZX- plane i.e., $y = 0$ $\therefore 3\lambda + 1 = 0$ $\lambda = -\frac{1}{3}$ $\therefore$ Point is $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$	1 1 ½ ½ 1 4
14.	**Case-Study Based Question** At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. Such a coin is unbiased with equal probabilities of getting head and tail. Based on the above information, answer the following questions : (a) If such a coin is tossed 2 times, then find the probability distribution of number of tails. (b) Find the probability of getting at least one head in three tosses of such a coin.	2 2
Sol.	(a) Let X denote the number of tails:	

	X	0	1	2	½
	$P(X)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	1½

(b) S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

P (at least one head) = 1–P (no head) — 1

$= 1 - \frac{1}{8}$

$= \frac{7}{8}$ — 1

2+2

* * *