

Series Q5QPS/5

Set - 1



प्रश्न-पत्र कोड  
Q.P. Code **65/5/1**

अनुक्रमांक  
Roll No.

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परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains 23 printed pages.
- Please check that this question paper contains 38 questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please write down the serial number of the question in the answer-book before attempting it.
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.



गणित

MATHEMATICS



निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

65/5/1/22/Q5QPS

**208 A**

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P.T.O.

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

- (i) इस प्रश्न-पत्र में कुल 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
- (ii) यह प्रश्न-पत्र पाँच खण्डों में विभाजित है – खण्ड-क, ख, ग, घ एवं ङ।
- (iii) खण्ड – क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित 1 अंक के प्रश्न हैं।
- (iv) खण्ड – ख में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के 2 अंकों के प्रश्न हैं।
- (v) खण्ड – ग में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के 3 अंकों के प्रश्न हैं।
- (vi) खण्ड – घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के 5 अंकों के प्रश्न हैं।
- (vii) खण्ड – ङ में प्रश्न संख्या 36 से 38 प्रकरण आधारित 4 अंकों के प्रश्न हैं।
- (viii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड – ख के 2 प्रश्नों में, खण्ड – ग के 3 प्रश्नों में, खण्ड – घ के 2 प्रश्नों में तथा खण्ड-ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
- (ix) कैल्कुलेटर का उपयोग वर्जित है।

**General Instructions :**

**Read the following instructions very carefully and strictly follow them :**

- (i) *This Question paper contains 38 questions. All questions are compulsory.*
- (ii) *Question paper is divided into FIVE Sections – Section A, B, C, D and E.*
- (iii) *In Section A – Questions Number 1 to 18 are Multiple Choice Questions (MCQs) type and Questions Number 19 & 20 are Assertion-Reason based questions of 1 mark each.*
- (iv) *In Section B – Questions Number 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.*
- (v) *In Section C – Questions Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.*
- (vi) *In Section D – Questions Number 32 to 35 are Long Answer (LA) type questions, carrying 5 marks each.*
- (vii) *In Section E – Questions Number 36 to 38 are case study based questions, carrying 4 marks each.*
- (viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section – B, 3 questions in Section – C, 2 questions in Section – D and 2 questions in Section – E.*
- (ix) *Use of calculators is NOT allowed.*

खण्ड – क

इस खण्ड में 20 बहुविकल्पी प्रश्न हैं, जिनमें प्रत्येक का 1 अंक है।

1.  $f(x) = x^2 - 4x + 5$  द्वारा परिभाषित फलन  $f : \mathbb{R} \rightarrow \mathbb{R}$

- (A) एकैकी है, परन्तु आच्छादक नहीं है। (B) आच्छादक है, परन्तु एकैकी नहीं है।  
(C) एकैकी तथा आच्छादक दोनों हैं। (D) न तो एकैकी और न ही आच्छादक है।

2. यदि  $A = \begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$  एक विषम-सममित आव्यूह है, तो  $2a - (b + c)$  का मान है :

- (A) 0 (B) 1  
(C) -10 (D) 10

3. यदि A, कोटि 3 का एक वर्ग आव्यूह है तथा  $|\text{adj} \cdot A| = 8$  है, तो  $|A^T|$  का मान है :

- (A)  $\sqrt{2}$  (B)  $-\sqrt{2}$   
(C) 8 (D)  $2\sqrt{2}$

4. यदि आव्यूह  $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$  का व्युत्क्रम आव्यूह  $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$  है, तो  $\lambda$  का मान है :

- (A) -4 (B) 1  
(C) 3 (D) 4

5. यदि  $[x \ 2 \ 0] \begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = [3 \ 1] \begin{bmatrix} -2 \\ x \end{bmatrix}$  है, तो  $x$  का मान है :

- (A) -1 (B) 0  
(C) 1 (D) 2

## SECTION – A

This section has **20** multiple choice questions of **1** mark each.

1. A function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined as  $f(x) = x^2 - 4x + 5$  is :
- (A) injective but not surjective.      (B) surjective but not injective.  
(C) both injective and surjective.      (D) neither injective nor surjective.

2. If  $A = \begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$  is a skew-symmetric matrix, then the value of  $2a - (b + c)$

is :

- (A) 0      (B) 1  
(C) -10      (D) 10
3. If A is a square matrix of order 3 such that the value of  $|\text{adj} \cdot A| = 8$ , then the value of  $|A^T|$  is :
- (A)  $\sqrt{2}$       (B)  $-\sqrt{2}$   
(C) 8      (D)  $2\sqrt{2}$

4. If inverse of matrix  $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$  is the matrix  $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$ , then value of  $\lambda$

is :

- (A) -4      (B) 1  
(C) 3      (D) 4
5. If  $[x \ 2 \ 0] \begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = [3 \ 1] \begin{bmatrix} -2 \\ x \end{bmatrix}$ , then value of  $x$  is :
- (A) -1      (B) 0  
(C) 1      (D) 2

6. यदि  $A = [a_{ij}]$  एक  $2 \times 2$  आव्यूह है, जिसमें  $a_{ij} = \text{maximum}(i, j) - \text{minimum}(i, j)$  द्वारा प्रदत्त हैं, तो आव्यूह  $A^2$  है :

(A)  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(B)  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C)  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(D)  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

7. यदि  $xe^y = 1$  है, तो  $x = 1$  पर  $\frac{dy}{dx}$  का मान है :

(A)  $-1$

(B)  $1$

(C)  $-e$

(D)  $-\frac{1}{e}$

8.  $e^{\sin^2 x}$  का  $\cos x$  के सापेक्ष अवकलज है :

(A)  $\sin x e^{\sin^2 x}$

(B)  $\cos x e^{\sin^2 x}$

(C)  $-2 \cos x e^{\sin^2 x}$

(D)  $-2 \sin^2 x \cos x e^{\sin^2 x}$

9. फलन  $f(x) = \frac{x}{2} + \frac{2}{x}$  का स्थानीय निम्नतम  $x$  के जिस मान पर है, वह है :

(A)  $2$

(B)  $1$

(C)  $0$

(D)  $-2$

10. एक वक्र  $y = 7x - x^3$  प्रदत्त है तथा  $x$ , 2 इकाई प्रति से. की दर से बढ़ रहा है। वक्र की ढाल के बदलने की दर, जब  $x = 5$  है, है

(A)  $-60$  इकाई/से.

(B)  $60$  इकाई/से.

(C)  $-70$  इकाई/से.

(D)  $-140$  इकाई/से.

11.  $\int \frac{1}{x(\log x)^2} dx$  बराबर है :

(A)  $2 \log(\log x) + c$

(B)  $-\frac{1}{\log x} + c$

(C)  $\frac{(\log x)^3}{3} + c$

(D)  $\frac{3}{(\log x)^3} + c$

6. Find the matrix  $A^2$ , where  $A = [a_{ij}]$  is a  $2 \times 2$  matrix whose elements are given by  $a_{ij} = \text{maximum}(i, j) - \text{minimum}(i, j)$  :

(A)  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(B)  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C)  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(D)  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

7. If  $xe^y = 1$ , then the value of  $\frac{dy}{dx}$  at  $x = 1$  is :

(A)  $-1$

(B)  $1$

(C)  $-e$

(D)  $-\frac{1}{e}$

8. Derivative of  $e^{\sin^2 x}$  with respect to  $\cos x$  is :

(A)  $\sin x e^{\sin^2 x}$

(B)  $\cos x e^{\sin^2 x}$

(C)  $-2 \cos x e^{\sin^2 x}$

(D)  $-2 \sin^2 x \cos x e^{\sin^2 x}$

9. The function  $f(x) = \frac{x}{2} + \frac{2}{x}$  has a local minima at  $x$  equal to :

(A)  $2$

(B)  $1$

(C)  $0$

(D)  $-2$

10. Given a curve  $y = 7x - x^3$  and  $x$  increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when  $x = 5$  is :

(A)  $-60$  units/sec

(B)  $60$  units/sec

(C)  $-70$  units/sec

(D)  $-140$  units/sec

11.  $\int \frac{1}{x(\log x)^2} dx$  is equal to :

(A)  $2 \log(\log x) + c$

(B)  $-\frac{1}{\log x} + c$

(C)  $\frac{(\log x)^3}{3} + c$

(D)  $\frac{3}{(\log x)^3} + c$

12.  $\int_{-1}^1 x|x| dx$  का मान है :

- (A)  $\frac{1}{6}$  (B)  $\frac{1}{3}$   
 (C)  $-\frac{1}{6}$  (D) 0

13. वक्र  $y^2 = 4x$ , X - अक्ष तथा  $x = 0$  और  $x = 1$  के बीच घिरे क्षेत्र का क्षेत्रफल है :

- (A)  $\frac{2}{3}$  (B)  $\frac{8}{3}$   
 (C) 3 (D)  $\frac{4}{3}$

14. अवकल समीकरण  $\frac{d^4y}{dx^4} - \sin\left(\frac{d^2y}{dx^2}\right) = 5$  की कोटि है :

- (A) 4 (B) 3  
 (C) 2 (D) परिभाषित नहीं है।

15.  $\vec{p}$  तथा  $\vec{q}$  क्रमशः बिंदुओं P तथा Q के स्थिति सदिश हैं। बिंदु R रेखाखण्ड PQ को 3 : 1 के अनुपात में विभाजित करता है तथा बिंदु S रेखाखण्ड PR का मध्य बिंदु है। S का स्थिति सदिश है :

- (A)  $\frac{\vec{p} + 3\vec{q}}{4}$  (B)  $\frac{\vec{p} + 3\vec{q}}{8}$   
 (C)  $\frac{5\vec{p} + 3\vec{q}}{4}$  (D)  $\frac{5\vec{p} + 3\vec{q}}{8}$

16. रेखा  $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$  द्वारा Y-अक्ष की धनात्मक दिशा से बनाया गया कोण है :

- (A)  $\frac{5\pi}{6}$  (B)  $\frac{3\pi}{4}$   
 (C)  $\frac{5\pi}{4}$  (D)  $\frac{7\pi}{4}$

12. The value of  $\int_{-1}^1 x |x| dx$  is :
- (A)  $\frac{1}{6}$  (B)  $\frac{1}{3}$   
 (C)  $-\frac{1}{6}$  (D) 0
13. Area of the region bounded by curve  $y^2 = 4x$  and the X-axis between  $x = 0$  and  $x = 1$  is :
- (A)  $\frac{2}{3}$  (B)  $\frac{8}{3}$   
 (C) 3 (D)  $\frac{4}{3}$
14. The order of the differential equation  $\frac{d^4y}{dx^4} - \sin\left(\frac{d^2y}{dx^2}\right) = 5$  is :
- (A) 4 (B) 3  
 (C) 2 (D) not defined
15. The position vectors of points P and Q are  $\vec{p}$  and  $\vec{q}$  respectively. The point R divides line segment PQ in the ratio 3 : 1 and S is the mid-point of line segment PR. The position vector of S is :
- (A)  $\frac{\vec{p} + 3\vec{q}}{4}$  (B)  $\frac{\vec{p} + 3\vec{q}}{8}$   
 (C)  $\frac{5\vec{p} + 3\vec{q}}{4}$  (D)  $\frac{5\vec{p} + 3\vec{q}}{8}$
16. The angle which the line  $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$  makes with the positive direction of Y-axis is :
- (A)  $\frac{5\pi}{6}$  (B)  $\frac{3\pi}{4}$   
 (C)  $\frac{5\pi}{4}$  (D)  $\frac{7\pi}{4}$

17. बिंदु  $(1, -3, 2)$  से होकर जाने वाली तथा रेखा  $\vec{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$  के समांतर एक रेखा का कार्तीय समीकरण है :

(A)  $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$

(B)  $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$

(C)  $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$

(D)  $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

18. यदि A और B दो घटनाएँ इस प्रकार हैं कि  $P(A/B) = P(B/A) \neq 0$ , तब :

(A)  $A \subset B$ , परन्तु  $A \neq B$

(B)  $A = B$

(C)  $A \cap B = \phi$

(D)  $P(A) = P(B)$

### अभिकथन – तर्क आधारित प्रश्न

निर्देश : प्रश्न 19 तथा 20 प्रत्येक में एक अभिकथन (A) के बाद एक तर्क (R) दिया गया है। निम्न विकल्पों में से सही उत्तर चुनिए :

(A) अभिकथन (A) तथा तर्क (R) दोनों सत्य हैं। तर्क (R) अभिकथन (A) की व्याख्या करता है।

(B) अभिकथन (A) तथा तर्क (R) दोनों सत्य हैं। तर्क (R) अभिकथन (A) की व्याख्या नहीं करता।

(C) अभिकथन (A) सत्य है, परन्तु तर्क (R) असत्य है।

(D) अभिकथन (A) असत्य है, जबकि तर्क (R) सत्य है।

19. अभिकथन (A) :  $y = \cos^{-1}(x)$  का प्रांत  $[-1, 1]$  है।

तर्क (R) :  $y = \cos^{-1}(x)$  की मुख्य शाखा की परिसर  $[0, \pi] - \left\{ \frac{\pi}{2} \right\}$  है।

17. The Cartesian equation of the line passing through the point  $(1, -3, 2)$  and parallel to the line :

$$\vec{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k} \text{ is}$$

- (A)  $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$                       (B)  $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$
- (C)  $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$                       (D)  $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

18. If A and B are events such that  $P(A/B) = P(B/A) \neq 0$ , then :

- (A)  $A \subset B$ , but  $A \neq B$                       (B)  $A = B$
- (C)  $A \cap B = \phi$                                   (D)  $P(A) = P(B)$

### Assertion – Reason Based Questions

**Direction :** In questions numbers **19** and **20**, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the following options :

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** Domain of  $y = \cos^{-1}(x)$  is  $[-1, 1]$ .

**Reason (R) :** The range of the principal value branch of  $y = \cos^{-1}(x)$  is  $[0, \pi] - \left\{ \frac{\pi}{2} \right\}$ .

20. अभिकथन (A) : सदिश

$$\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$$

$$\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k} \text{ तथा}$$

$$\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$$

एक समकोण त्रिभुज की भुजाओं को निरूपित करते हैं।

तर्क (R) : तीन शून्येतर सदिश जिनमें से कोई दो सरेख नहीं हैं, एक त्रिभुज बनाते हैं, यदि उनका परिणाम शून्य सदिश हो अथवा किन्हीं दो सदिशों का योग तीसरे सदिश के समान हो।

### खण्ड – ख

इस खण्ड में 5 अति लघु-उत्तरीय प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।

21. यदि  $\sin^{-1}\left[k \tan\left(2 \cos^{-1} \frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}$  है, तो k का मान ज्ञात कीजिए।

22. (a) सत्यापित कीजिए कि  $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$  द्वारा परिभाषित फलन f,  $x = 0$  पर संतत

है या नहीं।

### अथवा

(b)  $f(x) = |x - 5|$  द्वारा परिभाषित फलन f की  $x = 5$  पर अवकलनीयता की जाँच कीजिए।

23. एक वृत्त का क्षेत्रफल  $2 \text{ cm}^2/\text{sec}$  की एकसमान दर से बढ़ रहा है। ज्ञात कीजिए कि त्रिज्या  $r = 5 \text{ cm}$  पर इसकी परिधि के बढ़ने की दर क्या है?

20. **Assertion (A)** : The vectors

$$\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$$

$$\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$$

represent the sides of a right angled triangle.

**Reason (R)** : Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.

### SECTION – B

This section has 5 Very Short Answer questions of 2 marks each.

21. Find value of k if

$$\sin^{-1}\left[k \tan\left(2 \cos^{-1} \frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}.$$

22. (a) Verify whether the function f defined by

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is continuous at  $x = 0$  or not.

**OR**

(b) Check for differentiability of the function f defined by  $f(x) = |x - 5|$ , at the point  $x = 5$ .

23. The area of the circle is increasing at a uniform rate of 2 cm<sup>2</sup>/sec. How fast is the circumference of the circle increasing when the radius  $r = 5$  cm ?

24. (a) ज्ञात कीजिए :  $\int \cos^3 x e^{\log \sin x} dx$

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(b) ज्ञात कीजिए :  $\int \frac{1}{5 + 4x - x^2} dx$

25. बिंदु (2, 3, -5) से होकर जाने वाली तथा निर्देशांक अक्षों से समान कोण बनाने वाली रेखा का सदिश समीकरण ज्ञात कीजिए ।

खण्ड – ग

इस खण्ड में 6 लघु उत्तरीय प्रश्न हैं । प्रत्येक प्रश्न के 3 अंक हैं ।

26. (a) यदि  $(\cos x)^y = (\cos y)^x$  है, तो  $\frac{dy}{dx}$  ज्ञात कीजिए ।

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(b) यदि  $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$  है, तो सिद्ध कीजिए कि  $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

27. यदि  $x = a \sin^3 \theta$ ,  $y = b \cos^3 \theta$  है, तो  $\theta = \frac{\pi}{4}$  पर  $\frac{d^2y}{dx^2}$  ज्ञात कीजिए :

28. (a) मान ज्ञात कीजिए :  $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$

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(b) ज्ञात कीजिए :  $\int \frac{2x+1}{(x+1)^2(x-1)} dx$

24. (a) Find :  $\int \cos^3 x e^{\log \sin x} dx$

**OR**

(b) Find :  $\int \frac{1}{5 + 4x - x^2} dx$

25. Find the vector equation of the line passing through the point (2, 3, -5) and making equal angles with the co-ordinate axes.

### SECTION - C

There are **6** short answer questions in this section. Each is of **3** marks.

26. (a) Find  $\frac{dy}{dx}$ , if  $(\cos x)^y = (\cos y)^x$ .

**OR**

(b) If  $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ , prove that  $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ .

27. If  $x = a \sin^3 \theta$ ,  $y = b \cos^3 \theta$ , then find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{4}$ .

28. (a) Evaluate :  $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$

**OR**

(b) Find :  $\int \frac{2x+1}{(x+1)^2(x-1)} dx$

29. (a) अवकल समीकरण  $\frac{dy}{dx} - 2xy = 3x^2 e^{x^2}$ ;  $y(0) = 5$ , का विशिष्ट हल ज्ञात कीजिए ।

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- (b) निम्न अवकल समीकरण को हल कीजिए :

$$x^2 dy + y(x + y) dx = 0$$

30. सदिशों  $2\hat{i} - \hat{j} + \hat{k}$  तथा  $\hat{i} + \hat{j} - \hat{k}$  दोनों के लम्बवत ऐसा सदिश ज्ञात कीजिए जिसका परिमाण 4 इकाई हो । अतः अपने उत्तर की जाँच कीजिए ।

31. यादृच्छिक चर X का प्रायिकता बंटन नीचे दिया है, जहाँ a तथा b कोई स्थिरांक है :

<b>X</b>	1	2	3	4	5
<b>P(X)</b>	0.2	a	a	0.2	b

यदि माध्य  $E(X) = 3$  है तो a तथा b के मान ज्ञात कीजिए । अतः  $P(X \geq 3)$  ज्ञात कीजिए ।

खण्ड – घ

इस खण्ड में 4 दीर्घ-उत्तरीय प्रश्न हैं तथा प्रत्येक के 5 अंक हैं ।

32. (a) यदि  $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$  है, तो  $A^{-1}$  ज्ञात कीजिए । अतः निम्न समीकरण निकाय को हल

कीजिए :

$$x + 2y - 3z = 1$$

$$2x - 3z = 2$$

$$x + 2y = 3$$

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29. (a) Find the particular solution of the differential equation

$$\frac{dy}{dx} - 2xy = 3x^2 e^{-x^2}; y(0) = 5.$$

**OR**

- (b) Solve the following differential equation :

$$x^2 dy + y(x + y) dx = 0$$

30. Find a vector of magnitude 4 units perpendicular to each of the vectors

$$2\hat{i} - \hat{j} + \hat{k} \text{ and } \hat{i} + \hat{j} - \hat{k} \text{ and hence verify your answer.}$$

31. The random variable X has the following probability distribution where a and b are some constants :

<b>X</b>	1	2	3	4	5
<b>P(X)</b>	0.2	a	a	0.2	b

If the mean  $E(X) = 3$ , then find values of a and b and hence determine  $P(X \geq 3)$ .

### SECTION – D

There are 4 long answer questions in this section. Each question is of 5 marks.

32. (a) If  $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$ , then find  $A^{-1}$  and hence solve the following

system of equations :

$$x + 2y - 3z = 1$$

$$2x - 3z = 2$$

$$x + 2y = 3$$

**OR**

(b) आव्यूहों का गुणनफल  $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$  ज्ञात कीजिए :

अतः निम्न रेखीय समीकरण निकाय का हल ज्ञात कीजिए :

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

33. समाकलन से वक्र  $4x^2 + y^2 = 36$  से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

34. (a) बिंदु  $(2, 3, -8)$  से रेखा  $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$  पर डाले गए लंब के पाद के निर्देशांक ज्ञात कीजिए । अतः दिए गए बिंदु से दी गई रेखा पर खींचे लंब की लंबाई भी ज्ञात कीजिए ।

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(b) नीचे दी गई दो रेखाओं  $L_1$  तथा  $L_2$  के बीच की न्यूनतम दूरी ज्ञात कीजिए, जहाँ

$L_1$  : बिंदु  $(2, -1, 1)$  से होकर जाने वाली रेखा है जो  $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$  के समांतर है, तथा

$L_2$  :  $\vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$  है ।

35. निम्न रेखिक प्रोग्रामन समस्या को ग्राफ द्वारा हल कीजिए :

व्यवरोधों  $x + 2y \leq 12$

$$2x + y \leq 12$$

$$4x + 5y \geq 20$$

$x, y \geq 0$  के अंतर्गत  $Z = 60x + 40y$  का अधिकतमीकरण कीजिए ।

(b) Find the product of the matrices  $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$   $\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$  and

hence solve the system of linear equations :

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

33. Find the area of the region bounded by the curve  $4x^2 + y^2 = 36$  using integration.

34. (a) Find the co-ordinates of the foot of the perpendicular drawn from the point  $(2, 3, -8)$  to the line  $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ .

Also, find the perpendicular distance of the given point from the line.

**OR**

(b) Find the shortest distance between the lines  $L_1$  &  $L_2$  given below :

$L_1$  : The line passing through  $(2, -1, 1)$  and parallel to  $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$

$L_2$  :  $\vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$ .

35. Solve the following L.P.P. graphically :

Maximise  $Z = 60x + 40y$

Subject to  $x + 2y \leq 12$

$2x + y \leq 12$

$4x + 5y \geq 20$

$x, y \geq 0$

## खण्ड – ड

इस खण्ड में 3 प्रकरण आधारित प्रश्न हैं। प्रत्येक प्रश्न के 4 अंक हैं।

36. (a) एक विद्यालय के छात्रों को रेलवे की विरासत और उसके इतिहास के बारे में जानने के लिए रेलवे संग्रहालय ले जाया जाता है।



एक प्रदर्शनी में स्टेशन के पास ट्रैक पर कई रेल-लाइनों को दर्शाया गया है। माना  $L$  रेलवे ट्रैक पर स्थित सभी रेल-लाइनों का समुच्चय है तथा  $R = \{(l_1, l_2) : l_1, l_2 \text{ के समांतर है}\}$  द्वारा परिभाषित  $L$  पर संबंध है।

उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

- (i) ज्ञात कीजिए कि संबंध  $R$  सममित है या नहीं।
- (ii) ज्ञात कीजिए कि संबंध  $R$  संक्रामक है या नहीं।
- (iii) यदि रेलवे ट्रैक पर एक रेल-लाइन को समीकरण  $y = 3x + 2$  द्वारा निरूपित किया गया है, तो  $R$  में इससे संबंधित रेल-लाइनों के समुच्चय को ज्ञात कीजिए।

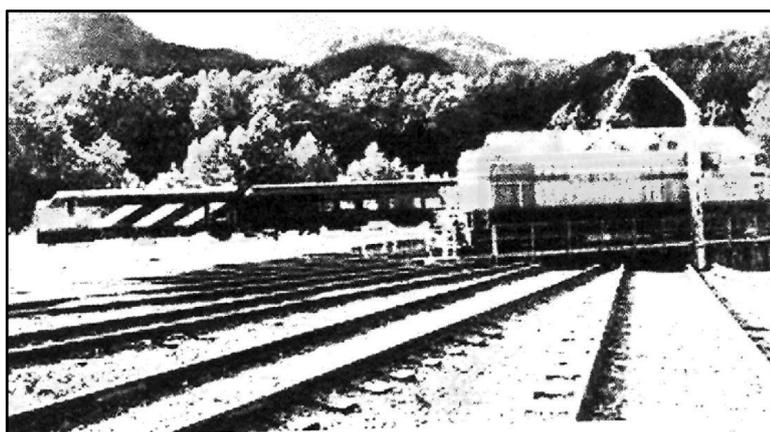
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- (b) माना एक संबंध  $S$ ,  $S = \{(l_1, l_2) : l_1, l_2 \text{ के लम्बवत है}\}$  द्वारा परिभाषित है, तो जाँच कीजिए कि क्या  $S$  सममित है तथा संक्रामक है।

## SECTION – E

In this section there are 3 case study questions of 4 marks each.

36. (a) Students of a school are taken to a railway museum to learn about railways heritage and its history.



An exhibit in the museum depicted many rail lines on the track near the railway station. Let  $L$  be the set of all rail lines on the railway track and  $R$  be the relation on  $L$  defined by

$$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$$

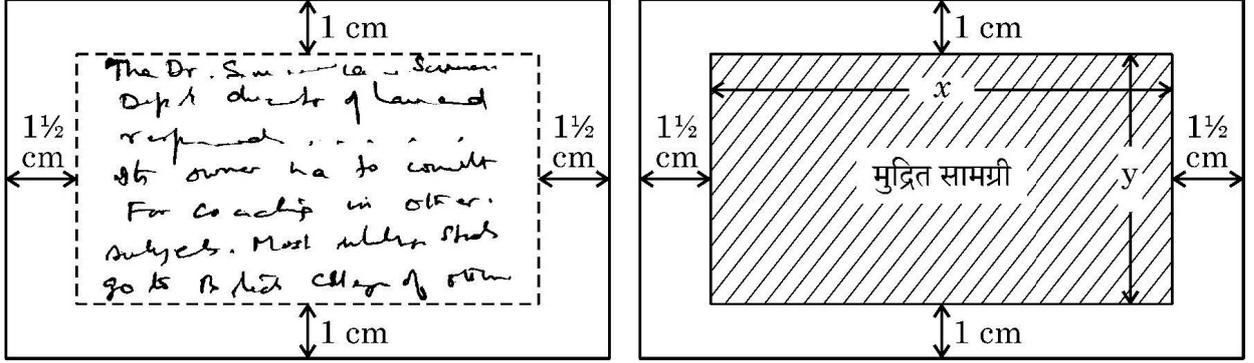
On the basis of the above information, answer the following questions :

- (i) Find whether the relation  $R$  is symmetric or not.
- (ii) Find whether the relation  $R$  is transitive or not.
- (iii) If one of the rail lines on the railway track is represented by the equation  $y = 3x + 2$ , then find the set of rail lines in  $R$  related to it.

**OR**

- (b) Let  $S$  be the relation defined by  $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$  check whether the relation  $S$  is symmetric and transitive.

37. एक आयताकार विजिटिंग कार्ड में 24 sq.cm. मुद्रित सामग्री होनी चाहिए। कार्ड के ऊपर और नीचे का मार्जिन 1 cm और बाएँ और दाएँ का मार्जिन  $1\frac{1}{2}$  cm होना चाहिए, जैसा दर्शाया गया है :



उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

- विजिटिंग कार्ड के क्षेत्रफल को  $x$  के पदों में व्यक्त कीजिए।
  - न्यूनतम क्षेत्रफल के लिए कार्ड की विमाएँ ज्ञात कीजिए।
38. एक डिपार्टमेंटल स्टोर अपने ग्राहकों से महीने में एक बार शुल्क लेने के लिए बिल भेजता है। पिछला अनुभव बताता है कि इसके 70% ग्राहक अपने पहले महीने का बिल समय पर चुकाते हैं। स्टोर ने यह भी पाया कि जो ग्राहक एक महीने में समय पर भुगतान करता है, उसके अगले महीने समय पर भुगतान करने की प्रायिकता 0.8 है और जो ग्राहक एक महीने में समय पर भुगतान नहीं करता है, उसके अगले महीने समय पर भुगतान करने की प्रायिकता 0.4 है।

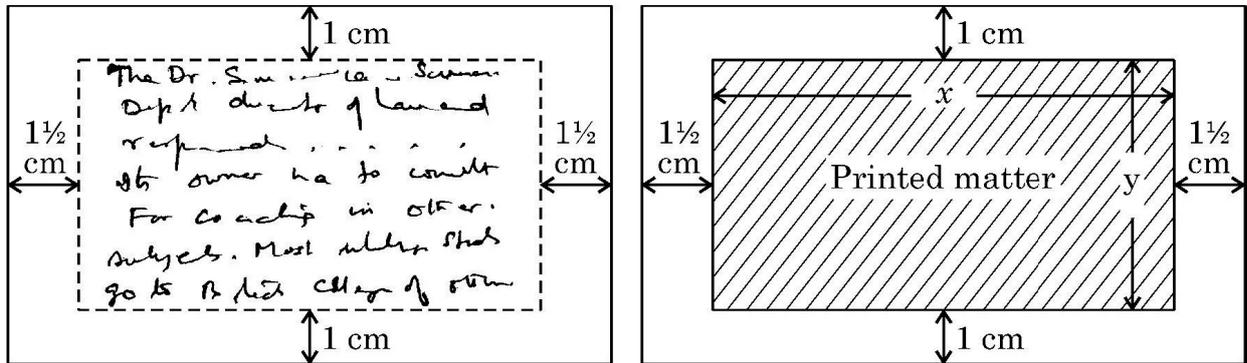
उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

- माना  $E_1$  तथा  $E_2$  क्रमशः ग्राहक द्वारा पहले महीने के बिल का समय पर भुगतान करने या न करने की घटना को दर्शाते हैं, तो  $P(E_1)$  तथा  $P(E_2)$  ज्ञात कीजिए।
- माना  $A$ , ग्राहक द्वारा दूसरे महीने का बिल समय पर देने की घटना को दर्शाता है, तो  $P(A|E_1)$  तथा  $P(A|E_2)$  ज्ञात कीजिए।
- ग्राहक द्वारा दूसरे महीने का बिल समय पर देने की प्रायिकता ज्ञात कीजिए।

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- ग्राहक द्वारा पहले महीने का बिल समय पर दिए जाने की प्रायिकता ज्ञात कीजिए, जहाँ दिया है कि उसने दूसरे महीने का बिल समय पर दिया है।

37. A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be  $1\frac{1}{2}$  cm as shown below :



On the basis of the above information, answer the following questions :

- (i) Write the expression for the area of the visiting card in terms of  $x$ .
  - (ii) Obtain the dimensions of the card of minimum area.
38. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions :

- (i) Let  $E_1$  and  $E_2$  respectively denote the event of customer paying or not paying the first month bill in time.  
Find  $P(E_1)$ ,  $P(E_2)$ .
- (ii) Let  $A$  denotes the event of customer paying second month's bill in time, then find  $P(A|E_1)$  and  $P(A|E_2)$ .
- (iii) Find the probability of customer paying second month's bill in time.

**OR**

- (iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.



**Marking Scheme**  
**Strictly Confidential**  
**(For Internal and Restricted use only)**  
**Senior School Certificate Examination, 2024**  
**MATHEMATICS PAPER CODE 65/5/1**

**General Instructions:**

<b>1</b>	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
<b>2</b>	<b>“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its’ leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC.”</b>
<b>3</b>	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. <b>However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.</b>
<b>4</b>	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
<b>5</b>	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
<b>6</b>	Evaluators will mark ( $\surd$ ) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right ( $\surd$ ) while evaluating which gives an impression that answer is correct and no marks are awarded. <b>This is most common mistake which evaluators are committing.</b>
<b>7</b>	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
<b>8</b>	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
<b>9</b>	<b><u>In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note “Extra Question”.</u></b>

10	<b><u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note “Extra Question”.</u></b>
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks _____(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- <ul style="list-style-type: none"> <li>● Leaving answer or part thereof unassessed in an answer book.</li> <li>● Giving more marks for an answer than assigned to it.</li> <li>● Wrong totaling of marks awarded on an answer.</li> <li>● Wrong transfer of marks from the inside pages of the answer book to the title page.</li> <li>● Wrong question wise totaling on the title page.</li> <li>● Wrong totaling of marks of the two columns on the title page.</li> <li>● Wrong grand total.</li> <li>● Marks in words and figures not tallying/not same.</li> <li>● Wrong transfer of marks from the answer book to online award list.</li> <li>● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)</li> <li>● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.</li> </ul>
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the “ <b>Guidelines for spot Evaluation</b> ” before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

## MARKING SCHEME – 65/5/1

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
<b>SECTION-A</b> (Question nos. 1 to 18 are Multiple choice Questions carrying 1 mark each)		
1.	A function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is : (A) injective but not surjective.      (B) surjective but not injective. (C) both injective and surjective.      (D) neither injective nor surjective.	
<b>Ans</b>	<b>(D) neither injective nor surjective</b>	<b>1</b>
2.	If $A = \begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of $2a - (b + c)$ is : (A) 0      (B) 1 (C) -10      (D) 10	
<b>Ans</b>	<b>(A) 0</b>	<b>1</b>
3.	If A is a square matrix of order 3 such that the value of $ \text{adj}\cdot A  = 8$ , then the value of $ A^T $ is : (A) $\sqrt{2}$ (B) $-\sqrt{2}$ (C) 8      (D) $2\sqrt{2}$	
<b>Ans</b>	<b>(D) <math>2\sqrt{2}</math></b>	<b>1</b>
4.	If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$ , then value of $\lambda$ is : (A) -4      (B) 1 (C) 3      (D) 4	
<b>Ans</b>	<b>(D) 4</b>	<b>1</b>

5.	<p>If <math>[x \ 2 \ 0] \begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = [3 \ 1] \begin{bmatrix} -2 \\ x \end{bmatrix}</math>, then value of <math>x</math> is :</p> <p>(A) <math>-1</math> (B) <math>0</math> (C) <math>1</math> (D) <math>2</math></p>	
<b>Ans</b>	<b>(A) <math>-1</math></b>	<b>1</b>
6.	<p>Find the matrix <math>A^2</math>, where <math>A = [a_{ij}]</math> is a <math>2 \times 2</math> matrix whose elements are given by <math>a_{ij} = \text{maximum}(i, j) - \text{minimum}(i, j)</math> :</p> <p>(A) <math>\begin{bmatrix} 0 &amp; 0 \\ 0 &amp; 0 \end{bmatrix}</math> (B) <math>\begin{bmatrix} 0 &amp; 1 \\ 1 &amp; 0 \end{bmatrix}</math> (C) <math>\begin{bmatrix} 1 &amp; 0 \\ 0 &amp; 1 \end{bmatrix}</math> (D) <math>\begin{bmatrix} 1 &amp; 1 \\ 1 &amp; 1 \end{bmatrix}</math></p>	
<b>Ans</b>	<b>(C) <math>\begin{bmatrix} 1 &amp; 0 \\ 0 &amp; 1 \end{bmatrix}</math></b>	<b>1</b>
7.	<p>If <math>xe^y = 1</math>, then the value of <math>\frac{dy}{dx}</math> at <math>x = 1</math> is :</p> <p>(A) <math>-1</math> (B) <math>1</math> (C) <math>-e</math> (D) <math>-\frac{1}{e}</math></p>	
<b>Ans</b>	<b>(A) <math>-1</math></b>	<b>1</b>
8.	<p>Derivative of <math>e^{\sin^2 x}</math> with respect to <math>\cos x</math> is :</p> <p>(A) <math>\sin x e^{\sin^2 x}</math> (B) <math>\cos x e^{\sin^2 x}</math> (C) <math>-2 \cos x e^{\sin^2 x}</math> (D) <math>-2 \sin^2 x \cos x e^{\sin^2 x}</math></p>	
<b>Ans</b>	<b>(C) <math>-2 \cos x e^{\sin^2 x}</math></b>	<b>1</b>
9.	<p>The function <math>f(x) = \frac{x}{2} + \frac{2}{x}</math> has a local minima at <math>x</math> equal to :</p> <p>(A) <math>2</math> (B) <math>1</math> (C) <math>0</math> (D) <math>-2</math></p>	
<b>Ans</b>	<b>(A) <math>2</math></b>	<b>1</b>





(Question Nos. 19 & 20 are Assertion-Reason based questions of 1 mark each)		
<b>Assertion – Reason Based Questions</b>		
<p><b>Direction :</b> In questions numbers <b>19</b> and <b>20</b>, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the following options :</p> <p>(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).</p> <p>(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).</p> <p>(C) Assertion (A) is true, but Reason (R) is false.</p> <p>(D) Assertion (A) is false, but Reason (R) is true.</p>		
<b>19.</b>	<p><b>Assertion (A) :</b> Domain of <math>y = \cos^{-1}(x)</math> is <math>[-1, 1]</math>.</p> <p><b>Reason (R) :</b> The range of the principal value branch of <math>y = \cos^{-1}(x)</math> is <math>[0, \pi] - \left\{ \frac{\pi}{2} \right\}</math>.</p>	
<b>Ans</b>	<b>(C) Assertion (A) is true, but Reason (R) is false</b>	<b>1</b>
<b>20.</b>	<p><b>Assertion (A) :</b> The vectors</p> $\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$ $\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$ $\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$ <p>represent the sides of a right angled triangle.</p> <p><b>Reason (R) :</b> Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.</p>	
<b>Ans</b>	<b>(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A)</b>	<b>1</b>
<b>SECTION-B</b>		
<b>(Question nos. 21 to 25 are very short Answer type questions carrying 2 marks each)</b>		
<b>21.</b>	<p>Find value of k if</p> $\sin^{-1} \left[ k \tan \left( 2 \cos^{-1} \frac{\sqrt{3}}{2} \right) \right] = \frac{\pi}{3}.$	



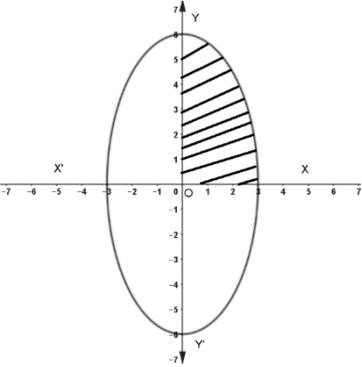
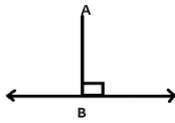
24.	<p>(a) Find : <math>\int \cos^3 x e^{\log \sin x} dx</math></p> <p style="text-align: center;"><b>OR</b></p> <p>(b) Find : <math>\int \frac{1}{5 + 4x - x^2} dx</math></p>	
Ans	<p>(a) <math>\int \cos^3 x e^{\log \sin x} dx = \int \cos^3 x \cdot \sin x dx</math>, Assuming <math>\cos x = t</math> and <math>\sin x dx = -dt</math></p> $= -\int t^3 dt$ $= -\frac{t^4}{4} + C = -\frac{\cos^4 x}{4} + C$ <p style="text-align: center;"><b>Or</b></p> <p>(b) <math>\int \frac{1}{5 + 4x - x^2} dx = \int \frac{1}{3^2 - (x-2)^2} dx</math></p> $= \frac{1}{6} \log \left  \frac{1+x}{5-x} \right  + C$	<p style="text-align: center;"><b>1</b></p> <p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;"><b>1</b></p> <p style="text-align: center;"><b>1</b></p>
25.	Find the vector equation of the line passing through the point (2, 3, -5) and making equal angles with the co-ordinate axes.	
Ans	<p>(a) <math>\cos \alpha = \cos \beta = \cos \gamma = l \Rightarrow l^2 + l^2 + l^2 = 1 \Rightarrow 3l^2 = 1, \therefore l = \frac{1}{\sqrt{3}}</math></p> <p>Direction cosines of the line are <math>\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \Rightarrow</math> the direction ratios are <b>1,1,1</b></p> <p><math>\therefore</math> Vector equation of the line is: <math>\vec{r} = 2\hat{i} + 3\hat{j} - 5\hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k})</math></p>	<p style="text-align: center;"><b>1</b></p> <p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;"><math>\frac{1}{2}</math></p>
<b>SECTION-C</b> <b>(Question nos. 26 to 31 are short Answer type questions carrying 3 marks each)</b>		
26.	<p>(a) Find <math>\frac{dy}{dx}</math>, if <math>(\cos x)^y = (\cos y)^x</math>.</p> <p style="text-align: center;"><b>OR</b></p> <p>(b) If <math>\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)</math>, prove that <math>\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}</math>.</p>	
Ans	<p>(a) Taking 'log' on both sides of <math>(\cos x)^y = (\cos y)^x</math>, we get</p> $y \log \cos x = x \log \cos y$	<b>1</b>

	$\Rightarrow \frac{dy}{dx} \log \cos x + y(-\tan x) = \log \cos y + x(-\tan y) \frac{dy}{dx}$ $\Rightarrow \frac{dy}{dx} = \frac{\log \cos y + y \tan x}{\log \cos x + x \tan y}$ <p style="text-align: center;"><b>Or</b></p> <p>(b) Let <math>x = \sin A, y = \sin B \therefore A = \sin^{-1} x, B = \sin^{-1} y</math></p> $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ $\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$ $\Rightarrow 2 \cos \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) = 2a \cos \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right)$ $\Rightarrow \cot \left( \frac{A-B}{2} \right) = a \Rightarrow A-B = 2 \cot^{-1} a, \therefore \sin^{-1} x - \sin^{-1} y = 2 \cot^{-1} a, \text{ differentiating with respect to 'x'}$ $\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$	<p><math>1\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p>1</p>
27.	If $x = a \sin^3 \theta, y = b \cos^3 \theta$ , then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$ .	
Ans	$\frac{dx}{d\theta} = 3a \sin^2 \theta \cos \theta, \frac{dy}{d\theta} = -3b \cos^2 \theta \sin \theta$ $\Rightarrow \frac{dy}{dx} = \frac{-3b \cos^2 \theta \sin \theta}{3a \sin^2 \theta \cos \theta} = -\frac{b}{a} \cot \theta$ $\Rightarrow \frac{d^2y}{dx^2} = \frac{b}{a} \operatorname{cosec}^2 \theta \frac{d\theta}{dx} = \frac{b}{a} \operatorname{cosec}^2 \theta \cdot \frac{1}{3a \sin^2 \theta \cos \theta} = \frac{b}{3a^2} \sec \theta \operatorname{cosec}^4 \theta$ $\Rightarrow \left. \frac{d^2y}{dx^2} \right]_{\theta=\frac{\pi}{4}} = \frac{4\sqrt{2}b}{3a^2}$	<p>1</p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p>
28.	<p>(a) Evaluate : <math>\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx</math></p> <p style="text-align: center;"><b>OR</b></p> <p>(b) Find : <math>\int \frac{2x+1}{(x+1)^2(x-1)} dx</math></p>	
Ans	<p>(a) Let <math>I = \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx</math> ----- (i)</p> $\Rightarrow I = \int_0^{\pi} \frac{e^{\cos(\pi-x)}}{e^{\cos(\pi-x)} + e^{-\cos(\pi-x)}} dx = \int_0^{\pi} \frac{e^{-\cos x}}{e^{-\cos x} + e^{\cos x}} dx$ ----- (ii) <p style="text-align: center;"><b>Adding (i) and (ii), we get</b></p> $2I = \int_0^{\pi} dx = x \Big _0^{\pi} = \pi, \therefore I = \frac{\pi}{2}$ <p style="text-align: center;"><b>Or</b></p>	<p><math>1\frac{1}{2}</math></p> <p><math>1\frac{1}{2}</math></p>

	<p>(b) <math>\int \frac{2x+1}{(x+1)^2(x-1)} dx = -\frac{3}{4} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{1}{(x+1)^2} dx + \frac{3}{4} \int \frac{1}{x-1} dx</math></p> $= -\frac{3}{4} \log x+1  - \frac{1}{2(x+1)} + \frac{3}{4} \log x-1  + C$ <p>or,</p> $= \frac{3}{4} \log \left  \frac{x-1}{x+1} \right  - \frac{1}{2(x+1)} + C$	$1\frac{1}{2}$  $1\frac{1}{2}$
29.	<p>(a) Find the particular solution of the differential equation</p> $\frac{dy}{dx} - 2xy = 3x^2 e^{x^2}; y(0) = 5.$ <p style="text-align: center;"><b>OR</b></p> <p>(b) Solve the following differential equation :</p> $x^2 dy + y(x+y) dx = 0$	
Ans	<p>(a) Given differential equation is a linear order differential equation with:</p> $P = -2x, Q = 3x^2 e^{x^2}$ $\text{Integrating Factor} = e^{\int -2x dx} = e^{-x^2}$ <p>The general solution is: <math>y \cdot e^{-x^2} = \int e^{-x^2} \cdot 3x^2 e^{x^2} dx + C \Rightarrow y \cdot e^{-x^2} = x^3 + C</math></p> <p>Putting <math>x=0, y=5</math>, we get, <math>C=5</math></p> <p><math>\therefore</math> The Particular solution is: <math>y \cdot e^{-x^2} = x^3 + 5</math> or <math>y = (x^3 + 5)e^{x^2}</math></p> <p style="text-align: center;"><b>Or</b></p> <p>(b) <math>x^2 dy + y(x+y) dx = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x} - \left(\frac{y}{x}\right)^2</math></p> $\text{Putting } \frac{y}{x} = v \Rightarrow y = vx, \frac{dy}{dx} = v + x \frac{dv}{dx}$ $v + x \frac{dv}{dx} = -v - v^2,$ <p>separating the variable and integrating</p> $\int \frac{1}{v^2 + 2v} dv = -\int \frac{1}{x} dx$ $\Rightarrow \int \frac{1}{(v+1)^2 - 1} dv = -\int \frac{1}{x} dx$ $\Rightarrow \frac{1}{2} \log \left  \frac{v}{v+2} \right  = \log \left  \frac{C}{x} \right $	$\frac{1}{2}$ 1 1 $\frac{1}{2}$  $\frac{1}{2}$  $\frac{1}{2}$  $\frac{1}{2}$

	The solution of the differential equation is, $\left  \frac{y}{y+2x} \right  = \frac{C^2}{x^2}$ or $x^2y = k(y+2x)$	$\frac{1}{2}$												
30.	Find a vector of magnitude 4 units perpendicular to each of the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$ and hence verify your answer.													
Ans	Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ , $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c}$ be the vector perpendicular to both $\vec{a}$ & $\vec{b}$ then, $\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 3\hat{j} + 3\hat{k}$ Let $\vec{d}$ the vector perpendicular to both the vectors $\vec{a}$ & $\vec{b}$ and having magnitude 4, $\vec{d} = 4\hat{c} = 2\sqrt{2}\hat{j} + 2\sqrt{2}\hat{k}$ (or $-2\sqrt{2}\hat{j} - 2\sqrt{2}\hat{k}$ ) Verification: $ \vec{d}  = \sqrt{8+8} = 4$ , $\vec{d} \cdot \vec{a} = \vec{d} \cdot \vec{b} = 0 \Rightarrow \vec{d} \perp \vec{a}$ and $\vec{d} \perp \vec{b}$	1 1 1												
31.	The random variable X has the following probability distribution where a and b are some constants : <table border="1" style="margin: 10px auto;"> <tbody> <tr> <td>X</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>P(X)</td> <td>0.2</td> <td>a</td> <td>a</td> <td>0.2</td> <td>b</td> </tr> </tbody> </table> If the mean $E(X) = 3$ , then find values of a and b and hence determine $P(X \geq 3)$ .	X	1	2	3	4	5	P(X)	0.2	a	a	0.2	b	
X	1	2	3	4	5									
P(X)	0.2	a	a	0.2	b									
Ans	$E(X) = 0.2 + 2a + 3a + 0.8 + 5b = 5a + 5b + 1$ $\sum p_i = 1 \Rightarrow 2a + b = 0.6$ $E(X) = 3 \Rightarrow 5a + 5b = 2$ and, solving the two equations, we get, $a = \frac{1}{5}, b = \frac{1}{5}$ $P(X \geq 3) = 1 - [P(X=1) + P(X=2)] = 1 - [0.2 + a] = 1 - \frac{2}{5} = \frac{3}{5}$	$\frac{1}{2}$ $\frac{1}{2}$ 1 1												
<b>SECTION-D</b> (Question nos. 32 to 35 are Long Answer type questions carrying 5 marks each)														
32.	(a) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$ , then find $A^{-1}$ and hence solve the following system of equations : $x + 2y - 3z = 1$ $2x - 3z = 2$ $x + 2y = 3$ OR													

	<p>(b) Find the product of the matrices <math>\begin{bmatrix} 1 &amp; 2 &amp; -3 \\ 2 &amp; 3 &amp; 2 \\ 3 &amp; -3 &amp; -4 \end{bmatrix} \begin{bmatrix} -6 &amp; 17 &amp; 13 \\ 14 &amp; 5 &amp; -8 \\ -15 &amp; 9 &amp; -1 \end{bmatrix}</math> and</p> <p>hence solve the system of linear equations :</p> $x + 2y - 3z = -4$ $2x + 3y + 2z = 2$ $3x - 3y - 4z = 11$	
Ans	<p>(a) <math> A  = 1(6) - 2(3) - 3(4) = -12 \neq 0, \therefore A^{-1}</math> exist</p> $\text{adj}A = \begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}$ $\therefore A^{-1} = -\frac{1}{12} \begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}$ <p>The given system of equations can be written as <math>AX = B, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}</math></p> $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{12} \begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1/2 \\ 2/3 \end{bmatrix}$ <p><math>\therefore</math> The solution of the given system of equations is: <math>x = 2, y = \frac{1}{2}, z = \frac{2}{3}</math></p> <p style="text-align: center;">Or</p> <p>(b) <math>\begin{bmatrix} 1 &amp; 2 &amp; -3 \\ 2 &amp; 3 &amp; 2 \\ 3 &amp; -3 &amp; -4 \end{bmatrix} \begin{bmatrix} -6 &amp; 17 &amp; 13 \\ 14 &amp; 5 &amp; -8 \\ -15 &amp; 9 &amp; -1 \end{bmatrix} = \begin{bmatrix} 67 &amp; 0 &amp; 0 \\ 0 &amp; 67 &amp; 0 \\ 0 &amp; 0 &amp; 67 \end{bmatrix}</math></p> $\Rightarrow \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}^{-1} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ <p>Solution of the system of equations is given by:</p> $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}^{-1} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix},$ <p><math>\therefore x = 3, y = -2, z = 1</math></p>	<p>1</p> <p>2</p> <p><math>\frac{1}{2}</math></p> <p><math>1\frac{1}{2}</math></p> <p><math>2\frac{1}{2}</math></p> <p>1</p> <p><math>1\frac{1}{2}</math></p>

<p>33.</p>	<p>Find the area of the region bounded by the curve <math>4x^2 + y^2 = 36</math> using integration.</p>	
<p>Ans</p>	<p>The given equation can be written as: <math>\frac{x^2}{9} + \frac{y^2}{36} = 1</math>, which is an ellipse.</p>  <p style="text-align: right;"><b>Correct Graph</b></p> <p>Area of the region bounded by the curve</p> $= 4 \times \frac{6}{3} \int_0^3 \sqrt{9-x^2} dx$ $= 8 \left[ \frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} \right]_0^3$ $= 18\pi$	<p>1</p> <p><math>1\frac{1}{2}</math></p> <p><math>1\frac{1}{2}</math></p> <p>1</p>
<p>34.</p>	<p>(a) Find the co-ordinates of the foot of the perpendicular drawn from the point <math>(2, 3, -8)</math> to the line <math>\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}</math>.</p> <p>Also, find the perpendicular distance of the given point from the line.</p> <p style="text-align: center;"><b>OR</b></p> <p>(b) Find the shortest distance between the lines <math>L_1</math> &amp; <math>L_2</math> given below :</p> <p><math>L_1</math> : The line passing through <math>(2, -1, 1)</math> and parallel to <math>\frac{x}{1} = \frac{y}{1} = \frac{z}{3}</math></p> <p><math>L_2</math> : <math>\vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}</math>.</p>	
<p>Ans</p>	<p>(a) The standard form of the equation of the line is <math>\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3}</math></p>  <p>Let foot of the perpendicular from the point <math>A(2, 3, -8)</math> to the given line be <math>B(-2\lambda + 4, 6\lambda, -3\lambda + 1)</math></p> <p>D-ratios of AB is: <math>-2\lambda + 2, 6\lambda - 3, -3\lambda + 9</math></p> <p>As AB is perpendicular to the given line: <math>-2(-2\lambda + 2) + 6(6\lambda - 3) - 3(-3\lambda + 9) = 0</math></p> $\Rightarrow \lambda = 1$ <p><math>\therefore</math> Foot of the perpendicular is: <math>B(2, 6, -2)</math></p> <p>Perpendicular distance = <math>AB = 3\sqrt{5}</math></p> <p style="text-align: center;"><b>Or</b></p>	<p><math>1\frac{1}{2}</math></p> <p><math>1\frac{1}{2}</math></p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>

(b) Equation of  $L_1$  :  $\vec{r} = 2\hat{i} - \hat{j} + \hat{k} + \lambda(\hat{i} + \hat{j} + 3\hat{k})$

Equation of  $L_2$  :  $\vec{r} = (\hat{i} + \hat{j} - 2\hat{k}) + \mu(2\hat{j} - \hat{k})$

Taking

$$\vec{a}_1 = 2\hat{i} - \hat{j} + \hat{k}, \quad \vec{b}_1 = \hat{i} + \hat{j} + 3\hat{k}$$

$$\vec{a}_2 = \hat{i} + \hat{j} - 2\hat{k}, \quad \vec{b}_2 = 2\hat{j} - \hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = -\hat{i} + 2\hat{j} - 3\hat{k}, \quad \vec{b}_1 \times \vec{b}_2 = -7\hat{i} + \hat{j} + 2\hat{k}$$

$$\text{Shortest Distance} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} = \frac{1}{\sqrt{6}}$$

$$\frac{1}{2}$$

$$\frac{1}{2}$$

$$\frac{1}{2} + 1$$

$$1\frac{1}{2}$$

35. Solve the following L.P.P. graphically :

Maximise  $Z = 60x + 40y$

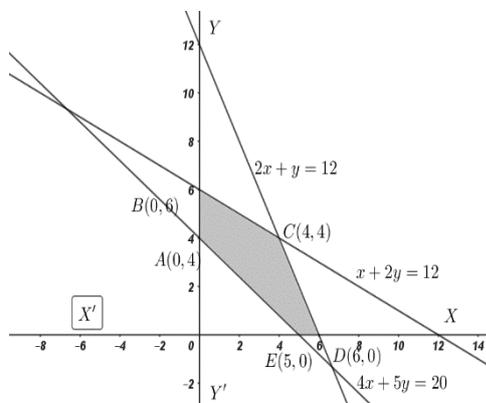
Subject to  $x + 2y \leq 12$

$$2x + y \leq 12$$

$$4x + 5y \geq 20$$

$$x, y \geq 0$$

Ans



Correct Graph

$$3\frac{1}{2}$$

Corner Points	Value of $Z = 60x + 40y$
A (0, 4)	$Z = 160$
B (0, 6)	$Z = 240$
C (4, 4)	$Z = 400$
D (6, 0)	$Z = 360$
E (5, 0)	$Z = 300$

$$\text{Max}(Z) = 400 \text{ at } x = 4, y = 4$$

1

$$\frac{1}{2}$$

## SECTION-E

(Question nos. 36 to 38 are source based/case based/passage based/integrated units of assessment questions carrying 4 marks each)

36.

- (a) Students of a school are taken to a railway museum to learn about railways heritage and its history.



An exhibit in the museum depicted many rail lines on the track near the railway station. Let  $L$  be the set of all rail lines on the railway track and  $R$  be the relation on  $L$  defined by

$$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$$

On the basis of the above information, answer the following questions :

- (i) Find whether the relation  $R$  is symmetric or not.
- (ii) Find whether the relation  $R$  is transitive or not.
- (iii) If one of the rail lines on the railway track is represented by the equation  $y = 3x + 2$ , then find the set of rail lines in  $R$  related to it.

**OR**

- (b) Let  $S$  be the relation defined by  $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$  check whether the relation  $S$  is symmetric and transitive.

Ans

(a) (i) Let  $(l_1, l_2) \in R \Rightarrow l_1 \parallel l_2 \Rightarrow l_2 \parallel l_1 \Rightarrow (l_2, l_1) \in R, \therefore R$  is a symmetric relation

1

(ii) Let  $(l_1, l_2), (l_2, l_3) \in R \Rightarrow l_1 \parallel l_2, l_2 \parallel l_3 \Rightarrow l_1 \parallel l_3 \Rightarrow (l_1, l_3) \in R, \therefore R$  is a transitive relation

1

(iii) The set is  $\{l : l \text{ is a line of type } y = 3x + c, c \in \mathbb{R}\}$

2

Or

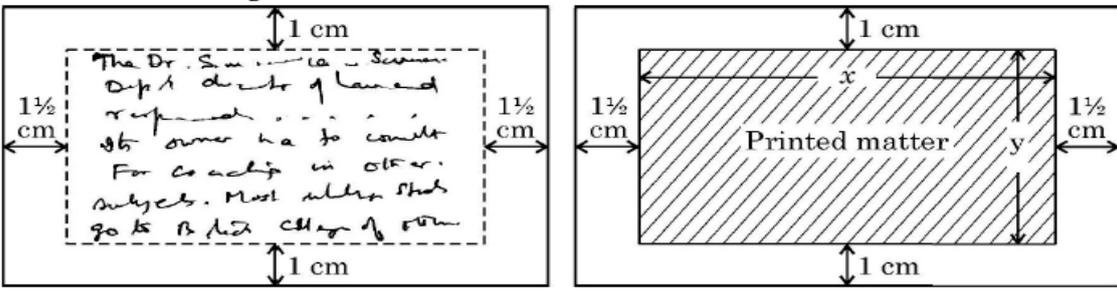
(b) Let  $(l_1, l_2) \in R \Rightarrow l_1 \perp l_2 \Rightarrow l_2 \perp l_1 \Rightarrow (l_2, l_1) \in R, \therefore R$  is a symmetric relation

2

Let  $(l_1, l_2), (l_2, l_3) \in R \Rightarrow l_1 \perp l_2, l_2 \perp l_3 \Rightarrow l_1 \parallel l_3 \Rightarrow (l_1, l_3) \notin R, \therefore R$  is not a transitive relation

2

**\*\* Due to printing error Part (a) or Part(b), both parts be taken as independent questions of 4 marks each**

37.	<p>A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be <math>1\frac{1}{2}</math> cm as shown below :</p>  <p>On the basis of the above information, answer the following questions :</p> <p>(i) Write the expression for the area of the visiting card in terms of <math>x</math>.</p> <p>(ii) Obtain the dimensions of the card of minimum area.</p>	
Ans	<p>(i) Let <math>A(x)</math> be the area of the visiting card then,</p> $\text{As } xy = 24, A(x) = (x+3)(y+2) = 2x + 3y + xy + 6 = 2x + \frac{72}{x} + 30$ <p>(ii) <math>A'(x) = 2 - \frac{72}{x^2}</math> and <math>A''(x) = \frac{144}{x^3}</math>,</p> <p>solving <math>A'(x) = 0 \Rightarrow x = 6</math> is the critical point.</p> $A''(6) = \frac{144}{6} > 0, \therefore \text{Area of the card is minimum at } x = 6, y = 4$ <p>The dimension of the card with minimum area is Length = 9 cm , Breadth = 6 cm</p>	<p>2</p> <p>1</p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>
38.	<p>A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.</p> <p>Based on the above information, answer the following questions :</p> <p>(i) Let <math>E_1</math> and <math>E_2</math> respectively denote the event of customer paying or not paying the first month bill in time. Find <math>P(E_1)</math>, <math>P(E_2)</math>.</p> <p>(ii) Let <math>A</math> denotes the event of customer paying second month's bill in time, then find <math>P(A E_1)</math> and <math>P(A E_2)</math>.</p> <p>(iii) Find the probability of customer paying second month's bill in time.</p> <p style="text-align: center;"><b>OR</b></p> <p>(iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.</p>	

<b>Ans</b>	(i) $P(E_1) = \frac{7}{10} = 0.7, P(E_2) = \frac{3}{10} = 0.3$	1
	(ii) $P(A E_1) = 0.8, P(A E_2) = 0.4$	1
	(iii) $P(A) = P(E_1) \cdot P(A E_1) + P(E_2) \cdot P(A E_2) = 0.7 \times 0.8 + 0.3 \times 0.4 = 0.68$ or $\frac{17}{25}$	2
	Or	
	(iii) $P(A) = \frac{P(E_1) \cdot P(A E_1)}{P(E_1) \cdot P(A E_1) + P(E_2) \cdot P(A E_2)} = \frac{14}{17}$	2