

Series Q5QPS/5

Set - 3



प्रश्न-पत्र कोड
Q.P. Code **65/5/3**

अनुक्रमांक
Roll No.

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परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains 23 printed pages.
- Please check that this question paper contains 38 questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please write down the serial number of the question in the answer-book before attempting it.
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.



गणित

MATHEMATICS



निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

65/5/3/22/Q5QPS

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P.T.O.

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

- (i) इस प्रश्न-पत्र में कुल 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
- (ii) यह प्रश्न-पत्र पाँच खण्डों में विभाजित है – खण्ड-क, ख, ग, घ एवं ङ।
- (iii) खण्ड – क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित 1 अंक के प्रश्न हैं।
- (iv) खण्ड – ख में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के 2 अंकों के प्रश्न हैं।
- (v) खण्ड – ग में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के 3 अंकों के प्रश्न हैं।
- (vi) खण्ड – घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के 5 अंकों के प्रश्न हैं।
- (vii) खण्ड – ङ में प्रश्न संख्या 36 से 38 प्रकरण आधारित 4 अंकों के प्रश्न हैं।
- (viii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड – ख के 2 प्रश्नों में, खण्ड – ग के 3 प्रश्नों में, खण्ड – घ के 2 प्रश्नों में तथा खण्ड-ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
- (ix) कैल्कुलेटर का उपयोग वर्जित है।

General Instructions :

Read the following instructions very carefully and strictly follow them :

- (i) *This Question paper contains 38 questions. All questions are compulsory.*
- (ii) *Question paper is divided into FIVE Sections – Section A, B, C, D and E.*
- (iii) *In Section A – Questions Number 1 to 18 are Multiple Choice Questions (MCQs) type and Questions Number 19 & 20 are Assertion-Reason based questions of 1 mark each.*
- (iv) *In Section B – Questions Number 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.*
- (v) *In Section C – Questions Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.*
- (vi) *In Section D – Questions Number 32 to 35 are Long Answer (LA) type questions, carrying 5 marks each.*
- (vii) *In Section E – Questions Number 36 to 38 are case study based questions, carrying 4 marks each.*
- (viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section – B, 3 questions in Section – C, 2 questions in Section – D and 2 questions in Section – E.*
- (ix) *Use of calculators is NOT allowed.*

खण्ड – क

इस खण्ड में 20 बहुविकल्पी प्रश्न हैं। प्रत्येक प्रश्न 1 अंक का है।

1. \vec{p} तथा \vec{q} क्रमशः बिंदुओं P तथा Q के स्थिति सदिश हैं। बिंदु R रेखाखण्ड PQ को 3 : 1 के अनुपात में विभाजित करता है तथा बिंदु S रेखाखण्ड PR का मध्य बिंदु है। S का स्थिति सदिश है :

- (A) $\frac{\vec{p} + 3\vec{q}}{4}$ (B) $\frac{\vec{p} + 3\vec{q}}{8}$
 (C) $\frac{5\vec{p} + 3\vec{q}}{4}$ (D) $\frac{5\vec{p} + 3\vec{q}}{8}$

2. आव्यूह $A = \begin{bmatrix} 2 & -1 & 1 \\ \lambda & 2 & 0 \\ 1 & -2 & 3 \end{bmatrix}$ के व्युत्क्रमणीय होने के लिए λ का मान है :

- (A) 0 (B) 10
 (C) $\mathbb{R} - \{10\}$ (D) $\mathbb{R} - \{-10\}$

3. रेखा $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ द्वारा Y-अक्ष की धनात्मक दिशा से बनाया गया कोण है :

- (A) $\frac{5\pi}{6}$ (B) $\frac{3\pi}{4}$
 (C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$

4. बिंदु (1, -3, 2) से होकर जाने वाली तथा रेखा $\vec{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$ के समांतर एक रेखा का कार्तीय समीकरण है :

- (A) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$ (B) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$
 (C) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$ (D) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

SECTION – A

There are **20** multiple choice questions of **1** mark each in this section.

1. The position vectors of points P and Q are \vec{p} and \vec{q} respectively. The point R divides line segment PQ in the ratio 3 : 1 and S is the mid-point of line segment PR. The position vector of S is :

(A) $\frac{\vec{p} + 3\vec{q}}{4}$

(B) $\frac{\vec{p} + 3\vec{q}}{8}$

(C) $\frac{5\vec{p} + 3\vec{q}}{4}$

(D) $\frac{5\vec{p} + 3\vec{q}}{8}$

2. For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ \lambda & 2 & 0 \\ 1 & -2 & 3 \end{bmatrix}$ to be invertible, the value of λ is :

(A) 0

(B) 10

(C) $\mathbb{R} - \{10\}$

(D) $\mathbb{R} - \{-10\}$

3. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y-axis is :

(A) $\frac{5\pi}{6}$

(B) $\frac{3\pi}{4}$

(C) $\frac{5\pi}{4}$

(D) $\frac{7\pi}{4}$

4. The Cartesian equation of the line passing through the point (1, -3, 2) and parallel to the line :

$\vec{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$ is

(A) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$

(B) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$

(C) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$

(D) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

5. यदि $A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ तथा $B = \begin{bmatrix} 4 & 0 \\ -1 & 1 \end{bmatrix}$ हैं, तो $A^2 = B$ के लिए x का मान है :
- (A) -2 (B) 2
(C) 2 या -2 (D) 4
6. एक वक्र $y = 7x - x^3$ प्रदत्त है तथा x , 2 इकाई प्रति से. की दर से बढ़ रहा है। वक्र की ढाल के बदलने की दर, जब $x = 5$ है, है
- (A) -60 इकाई/से. (B) 60 इकाई/से.
(C) -70 इकाई/से. (D) -140 इकाई/से.
7. माना कि $f(x) = \begin{vmatrix} x^2 & \sin x \\ p & -1 \end{vmatrix}$ है, जहाँ p एक अचर है। p का वह मान जिसके लिए $f'(0) = 1$ है, है :
- (A) \mathbb{R} (B) 1
(C) 0 (D) -1
8. यदि A और B दो घटनाएँ इस प्रकार हैं कि $P(A/B) = P(B/A) \neq 0$, तब :
- (A) $A \subset B$, परन्तु $A \neq B$ (B) $A = B$
(C) $A \cap B = \phi$ (D) $P(A) = P(B)$
9. $f(x) = x^2 - 4x + 5$ द्वारा परिभाषित फलन $f : \mathbb{R} \rightarrow \mathbb{R}$
- (A) एकैकी है, परन्तु आच्छादक नहीं है। (B) आच्छादक है, परन्तु एकैकी नहीं है।
(C) एकैकी तथा आच्छादक दोनों हैं। (D) न तो एकैकी और न ही आच्छादक है।
10. यदि A , कोटि 3 का एक वर्ग आव्यूह है तथा $|\text{adj} \cdot A| = 8$ है, तो $|A^T|$ का मान है :
- (A) $\sqrt{2}$ (B) $-\sqrt{2}$
(C) 8 (D) $2\sqrt{2}$

5. If $A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ -1 & 1 \end{bmatrix}$, then value of x for which $A^2 = B$ is :
- (A) -2 (B) 2
 (C) 2 or -2 (D) 4
6. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is :
- (A) -60 units/sec (B) 60 units/sec
 (C) -70 units/sec (D) -140 units/sec
7. Let $f(x) = \begin{vmatrix} x^2 & \sin x \\ p & -1 \end{vmatrix}$, where p is a constant. The value of p for which $f'(0) = 1$ is :
- (A) \mathbb{R} (B) 1
 (C) 0 (D) -1
8. If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then :
- (A) $A \subset B$, but $A \neq B$ (B) $A = B$
 (C) $A \cap B = \phi$ (D) $P(A) = P(B)$
9. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is :
- (A) injective but not surjective. (B) surjective but not injective.
 (C) both injective and surjective. (D) neither injective nor surjective.
10. If A is a square matrix of order 3 such that the value of $|\text{adj} \cdot A| = 8$, then the value of $|A^T|$ is :
- (A) $\sqrt{2}$ (B) $-\sqrt{2}$
 (C) 8 (D) $2\sqrt{2}$

11. यदि $\int_{-2}^3 x^2 dx = k \int_0^2 x^2 dx + \int_2^3 x^2 dx$ है, तो k का मान है :

- (A) 2 (B) 1
(C) 0 (D) $\frac{1}{2}$

12. $\int_1^e \log x dx$ का मान है :

- (A) 0 (B) 1
(C) e (D) $e \log e$

13. वक्र $y = \sqrt{x}$, Y-अक्ष तथा रेखाएँ $y = 0$, $y = 3$ के बीच घिरे क्षेत्र का क्षेत्रफल है :

- (A) $2\sqrt{3}$ (B) 27
(C) 9 (D) 3

14. निम्न अवकल समीकरण की कोटि :

$$\frac{d^3 y}{dx^3} + x \left(\frac{dy}{dx} \right)^5 = 4 \log \left(\frac{d^4 y}{dx^4} \right)$$

- (A) परिभाषित नहीं है। (B) 3 है।
(C) 4 है। (D) 5 है।

15. यदि आव्यूह $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ का व्युत्क्रम आव्यूह $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$ है, तो λ का मान है :

- (A) -4 (B) 1
(C) 3 (D) 4

16. यदि $A = [a_{ij}]$ एक 2×2 आव्यूह है, जिसमें $a_{ij} = \text{maximum}(i, j) - \text{minimum}(i, j)$ द्वारा प्रदत्त हैं, तो आव्यूह A^2 है :

- (A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

11. If $\int_{-2}^3 x^2 dx = k \int_0^2 x^2 dx + \int_2^3 x^2 dx$, then the value of k is :

- (A) 2 (B) 1
(C) 0 (D) $\frac{1}{2}$

12. The value of $\int_1^e \log x dx$ is :

- (A) 0 (B) 1
(C) e (D) $e \log e$

13. The area bounded by the curve $y = \sqrt{x}$, Y-axis and between the lines $y = 0$ and $y = 3$ is :

- (A) $2\sqrt{3}$ (B) 27
(C) 9 (D) 3

14. The order of the following differential equation

$$\frac{d^3y}{dx^3} + x \left(\frac{dy}{dx} \right)^5 = 4 \log \left(\frac{d^4y}{dx^4} \right) \text{ is :}$$

- (A) not defined (B) 3
(C) 4 (D) 5

15. If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ

is :

- (A) -4 (B) 1
(C) 3 (D) 4

16. Find the matrix A^2 , where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} = \text{maximum}(i, j) - \text{minimum}(i, j)$:

- (A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

17. $e^{\sin^2 x}$ का $\cos x$ के सापेक्ष अवकलज है :

- (A) $\sin x e^{\sin^2 x}$ (B) $\cos x e^{\sin^2 x}$
(C) $-2 \cos x e^{\sin^2 x}$ (D) $-2 \sin^2 x \cos x e^{\sin^2 x}$

18. फलन $f(x) = \frac{x}{2} + \frac{2}{x}$ का स्थानीय निम्नतम x के जिस मान पर है, वह है :

- (A) 2 (B) 1
(C) 0 (D) -2

अभिकथन – तर्क आधारित प्रश्न

निर्देश : प्रश्न 19 तथा 20 प्रत्येक में एक अभिकथन (A) के बाद एक तर्क (R) दिया गया है। निम्न विकल्पों में से सही उत्तर चुनिए :

- (A) अभिकथन (A) तथा तर्क (R) दोनों सत्य हैं। तर्क (R) अभिकथन (A) की व्याख्या करता है।
(B) अभिकथन (A) तथा तर्क (R) दोनों सत्य हैं। तर्क (R) अभिकथन (A) की व्याख्या नहीं करता।
(C) अभिकथन (A) सत्य है, परन्तु तर्क (R) असत्य है।
(D) अभिकथन (A) असत्य है, जबकि तर्क (R) सत्य है।

19. अभिकथन (A) : $y = \cos^{-1}(x)$ का प्रांत $[-1, 1]$ है।

तर्क (R) : $y = \cos^{-1}(x)$ की मुख्य शाखा की परिसर $[0, \pi] - \left\{ \frac{\pi}{2} \right\}$ है।

20. अभिकथन (A) : सदिश

$$\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$$

$$\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k} \text{ तथा}$$

$$\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$$

एक समकोण त्रिभुज की भुजाओं को निरूपित करते हैं।

तर्क (R) : तीन शून्येतर सदिश जिनमें से कोई दो सरेख नहीं हैं, एक त्रिभुज बनाते हैं, यदि उनका परिणाम शून्य सदिश हो अथवा किन्हीं दो सदिशों का योग तीसरे सदिश के समान हो।

17. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is :
- (A) $\sin x e^{\sin^2 x}$ (B) $\cos x e^{\sin^2 x}$
 (C) $-2 \cos x e^{\sin^2 x}$ (D) $-2 \sin^2 x \cos x e^{\sin^2 x}$
18. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to :
- (A) 2 (B) 1
 (C) 0 (D) -2

Assertion – Reason Based Questions

Direction : In questions numbers **19** and **20**, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the following options :

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
 (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
 (C) Assertion (A) is true, but Reason (R) is false.
 (D) Assertion (A) is false, but Reason (R) is true.
19. **Assertion (A) :** Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.
Reason (R) : The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{ \frac{\pi}{2} \right\}$.

20. **Assertion (A) :** The vectors
- $$\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$$
- $$\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$$
- $$\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$$
- represent the sides of a right angled triangle.
- Reason (R) :** Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.

खण्ड – ख

इस खण्ड में 5 अति लघु-उत्तरीय प्रश्न हैं। प्रत्येक प्रश्न के 2 अंक हैं।

21. सरल कीजिए : $\cos^{-1}x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right]$; $\frac{1}{2} \leq x \leq 1$

22. (a) ज्ञात कीजिए : $\int \cos^3 x e^{\log \sin x} dx$

अथवा

(b) ज्ञात कीजिए : $\int \frac{1}{5 + 4x - x^2} dx$

23. एक घन का पृष्ठीय क्षेत्रफल $72 \text{ cm}^2/\text{sec.}$ की दर से बढ़ रहा है। घन के आयतन के बढ़ने की दर ज्ञात कीजिए, जब घन के किनारे की लंबाई 3 cm है।

24. बिंदु $(2, 3, -5)$ से होकर जाने वाली तथा निर्देशांक अक्षों से समान कोण बनाने वाली रेखा का सदिश समीकरण ज्ञात कीजिए।

25. (a) सत्यापित कीजिए कि $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ द्वारा परिभाषित फलन f , $x = 0$ पर संतत

है या नहीं।

अथवा

(b) $f(x) = |x - 5|$ द्वारा परिभाषित फलन f की $x = 5$ पर अवकलनीयता की जाँच कीजिए।

SECTION – B

This section has 5 Very Short Answer questions of 2 marks each.

21. Simplify : $\cos^{-1}x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right]; \frac{1}{2} \leq x \leq 1$

22. (a) Find : $\int \cos^3 x e^{\log \sin x} dx$

OR

(b) Find : $\int \frac{1}{5 + 4x - x^2} dx$

23. The surface area of a cube increases at the rate of 72 cm²/sec. Find the rate of change of its volume, when the edge of the cube measures 3 cm.

24. Find the vector equation of the line passing through the point (2, 3, -5) and making equal angles with the co-ordinate axes.

25. (a) Verify whether the function f defined by

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0 & , x = 0 \end{cases}$$

is continuous at $x = 0$ or not.

OR

(b) Check for differentiability of the function f defined by $f(x) = |x - 5|$, at the point $x = 5$.

खण्ड – ग

इस खण्ड में 6 लघु-उत्तरीय प्रश्न हैं। प्रत्येक प्रश्न 3 अंकों का है।

26. (a) मान ज्ञात कीजिए : $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$

अथवा

(b) ज्ञात कीजिए : $\int \frac{2x+1}{(x+1)^2(x-1)} dx$

27. यदि $y = (\tan^{-1}x)^2$ है, तो दर्शाइए कि $(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1) \frac{dy}{dx} = 2$.

28. (a) अवकल समीकरण $\frac{dy}{dx} - 2xy = 3x^2 e^{x^2}$; $y(0) = 5$, का विशिष्ट हल ज्ञात कीजिए।

अथवा

(b) निम्न अवकल समीकरण को हल कीजिए :

$$x^2 dy + y(x + y) dx = 0$$

29. (a) यदि $(\cos x)^y = (\cos y)^x$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

अथवा

(b) यदि $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ है, तो सिद्ध कीजिए कि $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

SECTION – C

There are **6** short answer questions in this section. Each is of **3** marks.

26. (a) Evaluate : $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$

OR

(b) Find : $\int \frac{2x+1}{(x+1)^2(x-1)} dx$

27. If $y = (\tan^{-1}x)^2$, show that $(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1) \frac{dy}{dx} = 2$.

28. (a) Find the particular solution of the differential equation

$$\frac{dy}{dx} - 2xy = 3x^2 e^{x^2}; y(0) = 5.$$

OR

(b) Solve the following differential equation :

$$x^2 dy + y(x + y) dx = 0$$

29. (a) Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.

OR

(b) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

30. यदि $\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$ तथा $\vec{c} = \hat{i} + \hat{k}$ है, तो सदिश $(\vec{b} + \vec{c})$ का सदिश \vec{a} पर प्रक्षेप ज्ञात कीजिए।

31. एक कलश में 3 लाल तथा 2 सफेद कंचे हैं। इनमें से 2 कंचे एक-एक करके प्रतिस्थापना के साथ निकाले जाते हैं। निकाले गए सफेद कंचों की संख्या का प्रायिकता बंटन ज्ञात कीजिए। इस बंटन का माध्य भी ज्ञात कीजिए।

खण्ड – घ

इस खण्ड में 4 दीर्घ-उत्तरीय प्रश्न हैं। प्रत्येक प्रश्न के 5 अंक हैं।

32. समाकलन से वक्र $4x^2 + y^2 = 36$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

33. (a) बिंदु $(2, 3, -8)$ से रेखा $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ पर डाले गए लंब के पाद के निर्देशांक ज्ञात कीजिए। अतः दिए गए बिंदु से दी गई रेखा पर खींचे लंब की लंबाई भी ज्ञात कीजिए।

अथवा

(b) नीचे दी गई दो रेखाओं L_1 तथा L_2 के बीच की न्यूनतम दूरी ज्ञात कीजिए, जहाँ

L_1 : बिंदु $(2, -1, 1)$ से होकर जाने वाली रेखा है जो $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$ के समांतर है, तथा

L_2 : $\vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$ है।

34. (a) यदि $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए। अतः निम्न समीकरण निकाय को हल

कीजिए :

$$x + 2y - 3z = 1$$

$$2x - 3z = 2$$

$$x + 2y = 3$$

अथवा

30. Find the projection of vector $(\vec{b} + \vec{c})$ on vector \vec{a} , where $\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{k}$.
31. An urn contains 3 red and 2 white marbles. Two marbles are drawn one by one with replacement from the urn. Find the probability distribution of the number of white balls. Also, find the mean of the number of white balls drawn.

SECTION – D

There are 4 long answer questions in this section. Each question is of 5 marks.

32. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration.
33. (a) Find the co-ordinates of the foot of the perpendicular drawn from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.
- Also, find the perpendicular distance of the given point from the line.

OR

- (b) Find the shortest distance between the lines L_1 & L_2 given below :

L_1 : The line passing through $(2, -1, 1)$ and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$

L_2 : $\vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$.

34. (a) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$, then find A^{-1} and hence solve the following

system of equations :

$$x + 2y - 3z = 1$$

$$2x - 3z = 2$$

$$x + 2y = 3$$

OR

(b) आव्यूहों का गुणनफल $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ ज्ञात कीजिए :

अतः निम्न रेखीय समीकरण निकाय का हल ज्ञात कीजिए :

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

35. निम्न रेखिक प्रोग्रामन समस्या को ग्राफ द्वारा हल कीजिए :

व्यवरोधों $4x + y \geq 80;$

$$x + 5y \geq 115;$$

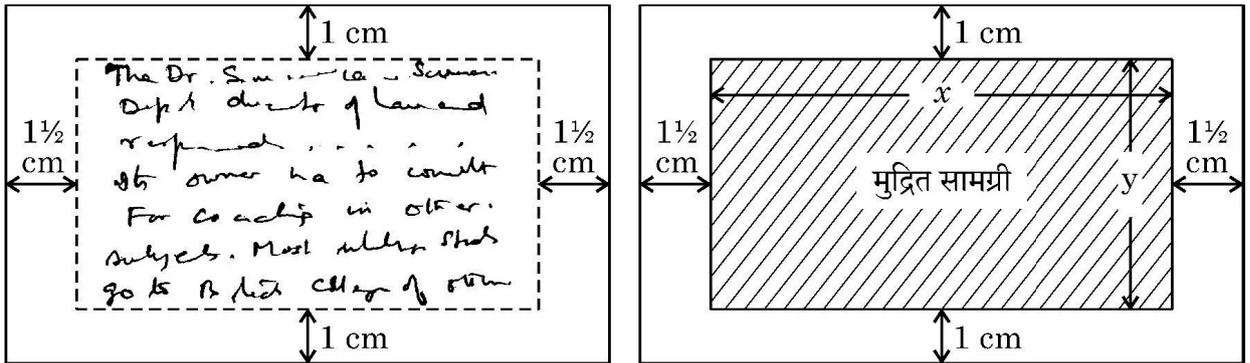
$$3x + 2y \leq 150$$

$$x, y \geq 0 \text{ के अंतर्गत } Z = 6x + 3y \text{ का न्यूनतमीकरण कीजिए ।}$$

खण्ड - ड

इस खण्ड में 3 प्रकरण आधारित प्रश्न हैं । प्रत्येक प्रश्न के 4 अंक हैं ।

36. एक आयताकार विजिटिंग कार्ड में 24 sq.cm. मुद्रित सामग्री होनी चाहिए । कार्ड के ऊपर और नीचे का मार्जिन 1 cm और बाएँ और दाएँ का मार्जिन $1\frac{1}{2}$ cm होना चाहिए, जैसा दर्शाया गया है :



उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

(i) विजिटिंग कार्ड के क्षेत्रफल को x के पदों में व्यक्त कीजिए ।

(ii) न्यूनतम क्षेत्रफल के लिए कार्ड की विमाएँ ज्ञात कीजिए ।

(b) Find the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$ $\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ and

hence solve the system of linear equations :

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

35. Solve the following L.P.P. graphically :

Minimise $Z = 6x + 3y$

Subject to constraints

$$4x + y \geq 80;$$

$$x + 5y \geq 115;$$

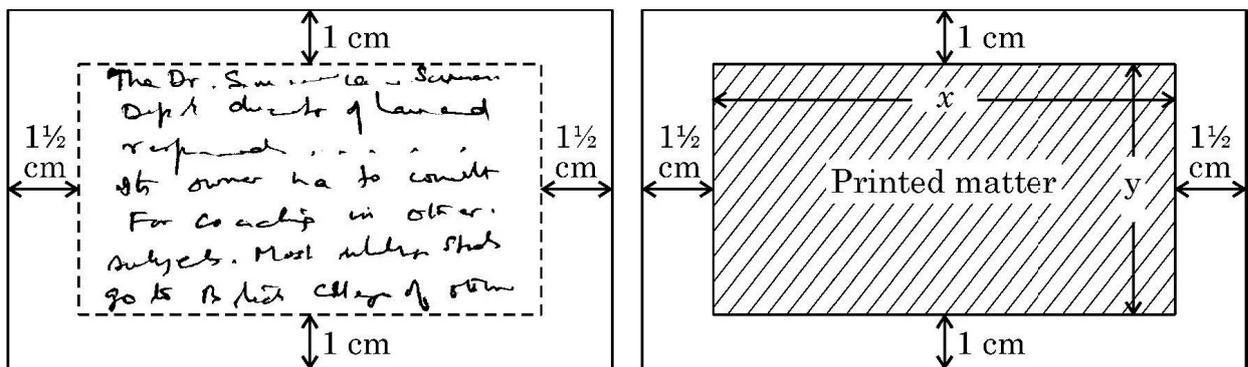
$$3x + 2y \leq 150$$

$$x, y \geq 0$$

SECTION – E

In this section there are 3 case study questions of 4 marks each.

36. A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be $1\frac{1}{2}$ cm as shown below :



On the basis of the above information, answer the following questions :

(i) Write the expression for the area of the visiting card in terms of x .

(ii) Obtain the dimensions of the card of minimum area.

37. एक डिपार्टमेंटल स्टोर अपने ग्राहकों से महीने में एक बार शुल्क लेने के लिए बिल भेजता है। पिछला अनुभव बताता है कि इसके 70% ग्राहक अपने पहले महीने का बिल समय पर चुकाते हैं। स्टोर ने यह भी पाया कि जो ग्राहक एक महीने में समय पर भुगतान करता है, उसके अगले महीने समय पर भुगतान करने की प्रायिकता 0.8 है और जो ग्राहक एक महीने में समय पर भुगतान नहीं करता है, उसके अगले महीने समय पर भुगतान करने की प्रायिकता 0.4 है।

उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

- (i) माना E_1 तथा E_2 क्रमशः ग्राहक द्वारा पहले महीने के बिल का समय पर भुगतान करने या न करने की घटना को दर्शाते हैं, तो $P(E_1)$ तथा $P(E_2)$ ज्ञात कीजिए।
- (ii) माना A, ग्राहक द्वारा दूसरे महीने का बिल समय पर देने की घटना को दर्शाता है, तो $P(A|E_1)$ तथा $P(A|E_2)$ ज्ञात कीजिए।
- (iii) ग्राहक द्वारा दूसरे महीने का बिल समय पर देने की प्रायिकता ज्ञात कीजिए।

अथवा

- (iii) ग्राहक द्वारा पहले महीने का बिल समय पर दिए जाने की प्रायिकता ज्ञात कीजिए, जहाँ दिया है कि उसने दूसरे महीने का बिल समय पर दिया है।

37. A departmental store sends bills to charge its customers once a month.

Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions :

(i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time.

Find $P(E_1)$, $P(E_2)$.

(ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$.

(iii) Find the probability of customer paying second month's bill in time.

OR

(iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

38. (a) एक विद्यालय के छात्रों को रेलवे की विरासत और उसके इतिहास के बारे में जानने के लिए रेलवे संग्रहालय ले जाया जाता है ।



एक प्रदर्शनी में स्टेशन के पास ट्रैक पर कई रेल-लाइनों को दर्शाया गया है । माना L रेलवे ट्रैक पर स्थित सभी रेल-लाइनों का समुच्चय है तथा $R = \{(l_1, l_2) : l_1, l_2 \text{ के समांतर है}\}$ द्वारा परिभाषित L पर संबंध है ।

उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

- (i) ज्ञात कीजिए कि संबंध R सममित है या नहीं ।
- (ii) ज्ञात कीजिए कि संबंध R संक्रामक है या नहीं ।
- (iii) यदि रेलवे ट्रैक पर एक रेल-लाइन को समीकरण $y = 3x + 2$ द्वारा निरूपित किया गया है, तो R में इससे संबंधित रेल-लाइनों के समुच्चय को ज्ञात कीजिए ।

अथवा

- (b) माना एक संबंध S , $S = \{(l_1, l_2) : l_1, l_2 \text{ के लम्बवत है}\}$ द्वारा परिभाषित है, तो जाँच कीजिए कि क्या S सममित है तथा संक्रामक है ।

38. (a) Students of a school are taken to a railway museum to learn about railways heritage and its history.



An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$$

On the basis of the above information, answer the following questions :

- (i) Find whether the relation R is symmetric or not.
- (ii) Find whether the relation R is transitive or not.
- (iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

OR

- (b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior School Certificate Examination, 2024
MATHEMATICS PAPER CODE 65/5/3

General Instructions:

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its’ leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC.”
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (\checkmark) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (\checkmark) while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	<u>In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note “Extra Question”.</u>

10	<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note “Extra Question”.</u>
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks _____(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- <ul style="list-style-type: none"> ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the “ Guidelines for spot Evaluation ” before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

8.	If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then : (A) $A \subset B$, but $A \neq B$ (B) $A = B$ (C) $A \cap B = \phi$ (D) $P(A) = P(B)$	
Ans	(D) $P(A) = P(B)$	1
9.	A function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is : (A) injective but not surjective. (B) surjective but not injective. (C) both injective and surjective. (D) neither injective nor surjective.	
Ans	(D) neither injective nor surjective	1
10.	If A is a square matrix of order 3 such that the value of $ \text{adj} \cdot A = 8$, then the value of $ A^T $ is : (A) $\sqrt{2}$ (B) $-\sqrt{2}$ (C) 8 (D) $2\sqrt{2}$	
Ans	(D) $2\sqrt{2}$	1
11.	If $\int_{-2}^3 x^2 dx = k \int_0^2 x^2 dx + \int_2^3 x^2 dx$, then the value of k is : (A) 2 (B) 1 (C) 0 (D) $\frac{1}{2}$	
Ans	(A) 2	1
12.	The value of $\int_1^e \log x dx$ is : (A) 0 (B) 1 (C) e (D) $e \log e$	
Ans	(B) 1	1

13.	<p>The area bounded by the curve $y = \sqrt{x}$, Y-axis and between the lines $y = 0$ and $y = 3$ is :</p> <p>(A) $2\sqrt{3}$ (B) 27 (C) 9 (D) 3</p>	
Ans	(C) 9	1
14.	<p>The order of the following differential equation $\frac{d^3y}{dx^3} + x\left(\frac{dy}{dx}\right)^5 = 4\log\left(\frac{d^4y}{dx^4}\right)$ is :</p> <p>(A) not defined (B) 3 (C) 4 (D) 5</p>	
Ans	(C) 4	1
15.	<p>If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is :</p> <p>(A) -4 (B) 1 (C) 3 (D) 4</p>	
Ans	(D) 4	1
16.	<p>Find the matrix A^2, where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} = \text{maximum}(i, j) - \text{minimum}(i, j)$:</p> <p>(A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$</p>	
Ans	(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	1

17.	Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is : (A) $\sin x e^{\sin^2 x}$ (B) $\cos x e^{\sin^2 x}$ (C) $-2 \cos x e^{\sin^2 x}$ (D) $-2 \sin^2 x \cos x e^{\sin^2 x}$	
Ans	(C) $-2 \cos x e^{\sin^2 x}$	1
18.	The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to : (A) 2 (B) 1 (C) 0 (D) -2	
Ans	(A) 2	1
(Question Nos. 19 & 20 are Assertion-Reason based questions of 1 mark each)		
Assertion – Reason Based Questions		
<p>Direction : In questions numbers 19 and 20, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the following options :</p> <p>(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.</p>		
19.	<p>Assertion (A) : Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.</p> <p>Reason (R) : The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{ \frac{\pi}{2} \right\}$.</p>	
Ans	(C) Assertion (A) is true, but Reason (R) is false	1

Ans	<p>(a) $\int \cos^3 x e^{\log \sin x} dx = \int \cos^3 x \cdot \sin x dx$, Assuming $\cos x = t$ and $\sin x dx = -dt$</p> $= -\int t^3 dt$ $= -\frac{t^4}{4} + C = -\frac{\cos^4 x}{4} + C$ <p style="text-align: center;">Or</p> <p>(b) $\int \frac{1}{5+4x-x^2} dx = \int \frac{1}{3^2 - (x-2)^2} dx$</p> $= \frac{1}{6} \log \left \frac{1+x}{5-x} \right + C$	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p>
23.	The surface area of a cube increases at the rate of $72 \text{ cm}^2/\text{sec}$. Find the rate of change of its volume, when the edge of the cube measures 3 cm.	
Ans	<p>Let edge of cube be 'x cm'</p> $S = 6x^2, \frac{dS}{dt} = 72 \text{ cm}^2/\text{sec} \Rightarrow 12x \frac{dx}{dt} = 72 \Rightarrow \frac{dx}{dt} = \frac{6}{x}$ $\text{Volume, } V = x^3 \Rightarrow \frac{dV}{dt} = 3x^2 \frac{dx}{dt} = 3 \times x^2 \times \frac{6}{x} = 18x, \left. \frac{dV}{dt} \right _{x=3} = 54 \text{ cm}^3/\text{sec}$ <p>\therefore Volume is increasing at the rate of $54 \text{ cm}^3/\text{sec}$</p>	<p>$\frac{1}{2} + \frac{1}{2}$</p> <p>$\frac{1}{2} + \frac{1}{2}$</p>
24.	Find the vector equation of the line passing through the point (2, 3, -5) and making equal angles with the co-ordinate axes.	
Ans	<p>(a) $\cos \alpha = \cos \beta = \cos \gamma = l \Rightarrow l^2 + l^2 + l^2 = 1 \Rightarrow 3l^2 = 1, \therefore l = \frac{1}{\sqrt{3}}$</p> <p>Direction cosines of the line are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \Rightarrow$ the direction ratios are 1,1,1</p> <p>\therefore Vector equation of line is: $\vec{r} = 2\hat{i} + 3\hat{j} - 5\hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k})$</p>	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>

25.	<p>(a) Verify whether the function f defined by</p> $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0 & , x = 0 \end{cases}$ <p>is continuous at $x = 0$ or not.</p> <p style="text-align: center;">OR</p> <p>(b) Check for differentiability of the function f defined by $f(x) = x - 5$, at the point $x = 5$.</p>	
Ans	<p>(a) $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} x \cdot \sin \frac{1}{x} = 0 \times \text{Finite value in } [-1, 1] = 0 = f(0)$</p> <p>$\therefore f(x)$ is a continuous function.</p> <p style="text-align: center;">Or</p> <p>(b) $\text{LHD} = \lim_{x \rightarrow 5^-} \frac{ x-5 - 0}{x-5} = \lim_{x \rightarrow 5^-} \frac{-(x-5)}{x-5} = -1$</p> <p>$\text{RHD} = \lim_{x \rightarrow 5^+} \frac{ x-5 - 0}{x-5} = \lim_{x \rightarrow 5^+} \frac{(x-5)}{x-5} = 1$</p> <p>$\text{LHD} \neq \text{RHD}, \therefore f(x)$ is not differentiable at $x = 5$</p>	<p>$1\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
<p>SECTION-C (Question nos. 26 to 31 are short Answer type questions carrying 3 marks each)</p>		
26.	<p>(a) Evaluate : $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$</p> <p style="text-align: center;">OR</p> <p>(b) Find : $\int \frac{2x+1}{(x+1)^2(x-1)} dx$</p>	
Ans	<p>(a) Let $I = \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$</p>	----- (i)

	$\Rightarrow I = \int_0^{\pi} \frac{e^{\cos(\pi-x)}}{e^{\cos(\pi-x)} + e^{-\cos(\pi-x)}} dx = \int_0^{\pi} \frac{e^{-\cos x}}{e^{-\cos x} + e^{\cos x}} dx \quad \text{----- (ii)}$ <p>Adding (i) and (ii), we get</p> $2I = \int_0^{\pi} dx = x \Big _0^{\pi} = \pi, \therefore I = \frac{\pi}{2}$ <p>Or</p>	$1\frac{1}{2}$ $1\frac{1}{2}$
	<p>(b) $\int \frac{2x+1}{(x+1)^2(x-1)} dx = -\frac{3}{4} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{1}{(x+1)^2} dx + \frac{3}{4} \int \frac{1}{x-1} dx$</p> $= -\frac{3}{4} \log x+1 - \frac{1}{2(x+1)} + \frac{3}{4} \log x-1 + C$ <p>or,</p> $= \frac{3}{4} \log \left \frac{x-1}{x+1} \right - \frac{1}{2(x+1)} + C$	$1\frac{1}{2}$ $1\frac{1}{2}$
27.	If $y = (\tan^{-1}x)^2$, show that $(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1) \frac{dy}{dx} = 2$.	
Ans	$y = (\tan^{-1} x)^2 \Rightarrow \frac{dy}{dx} = \frac{2 \tan^{-1} x}{1+x^2}$ $\Rightarrow (1+x^2) \frac{dy}{dx} = 2 \tan^{-1} x, \text{ differentiating with respect to 'x'}$ $\Rightarrow (1+x^2)^2 \frac{d^2y}{dx^2} + 2x(x^2+1) \frac{dy}{dx} = 2$	$1\frac{1}{2}$ $1\frac{1}{2}$
28.	<p>(a) Find the particular solution of the differential equation</p> $\frac{dy}{dx} - 2xy = 3x^2 e^{-x^2}; y(0) = 5.$ <p>OR</p> <p>(b) Solve the following differential equation :</p> $x^2 dy + y(x+y) dx = 0$	
Ans	<p>(a) Given differential equation is a linear order differential equation with:</p> $P = -2x, Q = 3x^2 e^{-x^2}$ <p>Integrating Factor = $e^{\int -2x dx} = e^{-x^2}$</p> <p>The general solution is : $y \cdot e^{-x^2} = \int e^{-x^2} \cdot 3x^2 e^{-x^2} dx + C \Rightarrow y \cdot e^{-x^2} = x^3 + C$</p>	$\frac{1}{2}$ 1 1

	<p>Putting $x=0, y=5$, we get, $C=5$</p> <p>\therefore The Particular solution is: $y \cdot e^{-x^2} = x^3 + 5$ or $y = (x^3 + 5)e^{x^2}$</p> <p>Or</p> <p>(b) $x^2 dy + y(x+y) dx = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x} - \left(\frac{y}{x}\right)^2$</p> <p>Putting $\frac{y}{x} = v \Rightarrow y = vx, \frac{dy}{dx} = v + x \frac{dv}{dx}$</p> $v + x \frac{dv}{dx} = -v - v^2,$ <p>separating the variable and integrating</p> $\int \frac{1}{v^2 + 2v} dv = -\int \frac{1}{x} dx$ $\Rightarrow \int \frac{1}{(v+1)^2 - 1} dv = -\int \frac{1}{x} dx$ $\Rightarrow \frac{1}{2} \log \left \frac{v}{v+2} \right = \log \left \frac{C}{x} \right $ <p>The solution of the differential equation is, $\left \frac{y}{y+2x} \right = \frac{C^2}{x^2}$ or $x^2 y = k(y+2x)$</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
29.	<p>(a) Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.</p> <p>OR</p> <p>(b) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.</p>	
Ans	<p>(a) Taking 'log' on both sides of $(\cos x)^y = (\cos y)^x$, we get</p> $y \log \cos x = x \log \cos y$ $\Rightarrow \frac{dy}{dx} \log \cos x + y(-\tan x) = \log \cos y + x(-\tan y) \frac{dy}{dx}$ $\Rightarrow \frac{dy}{dx} = \frac{\log \cos y + y \tan x}{\log \cos x + x \tan y}$ <p>Or</p> <p>(b) Let $x = \sin A, y = \sin B \therefore A = \sin^{-1} x, B = \sin^{-1} y$</p>	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>

	$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ $\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$ $\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ $\Rightarrow \cot\left(\frac{A-B}{2}\right) = a \Rightarrow A-B = 2\cot^{-1} a, \therefore \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a, \text{ differentiating with respect to 'x'}$ $\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$	1 $\frac{1}{2}$ 1								
30.	Find the projection of vector $(\vec{b} + \vec{c})$ on vector \vec{a} , where $\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{k}$.									
Ans	$\vec{b} + \vec{c} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ $\text{Projection of } (\vec{b} + \vec{c}) \text{ on } \vec{a} = \frac{(\vec{b} + \vec{c}) \cdot \vec{a}}{ \vec{a} } = \frac{4+6+2}{\sqrt{4+4+1}} = 4$	1 $1\frac{1}{2}$ $\frac{1}{2}$								
31.	An urn contains 3 red and 2 white marbles. Two marbles are drawn one by one with replacement from the urn. Find the probability distribution of the number of white balls. Also, find the mean of the number of white balls drawn.									
Ans	<p>Let X = Number of white marbles</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td>X</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>P(X)</td> <td>$\frac{9}{25}$</td> <td>$\frac{12}{25}$</td> <td>$\frac{4}{25}$</td> </tr> </tbody> </table> $\text{Mean} = 0 \times \frac{9}{25} + 1 \times \frac{12}{25} + 2 \times \frac{4}{25} = \frac{20}{25} = \frac{4}{5}$	X	0	1	2	P(X)	$\frac{9}{25}$	$\frac{12}{25}$	$\frac{4}{25}$	1 $1\frac{1}{2}$ $\frac{1}{2}$
X	0	1	2							
P(X)	$\frac{9}{25}$	$\frac{12}{25}$	$\frac{4}{25}$							

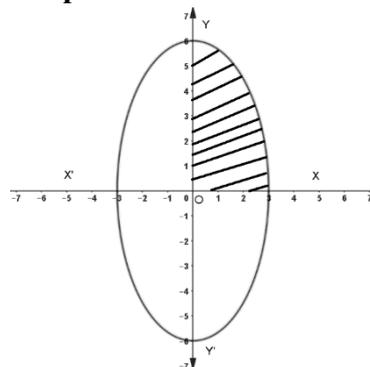
SECTION-D

(Question nos. 32 to 35 are Long Answer type questions carrying 5 marks each)

32. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration.

Ans The given equation can be written as: $\frac{x^2}{9} + \frac{y^2}{36} = 1$, which is an ellipse.

Graph of the curve is.



Area of the curve

$$\begin{aligned}
 &= 4 \times \frac{6}{3} \int_0^3 \sqrt{9-x^2} dx \\
 &= 8 \left[\frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} \right]_0^3 \\
 &= 18\pi
 \end{aligned}$$

Correct Graph

1

$1\frac{1}{2}$

$1\frac{1}{2}$

1

33. (a) Find the co-ordinates of the foot of the perpendicular drawn from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

Also, find the perpendicular distance of the given point from the line.

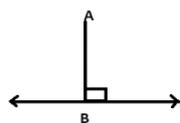
OR

- (b) Find the shortest distance between the lines L_1 & L_2 given below :

L_1 : The line passing through $(2, -1, 1)$ and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$

L_2 : $\vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$.

Ans (a) The standard form of the equation of the line is $\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3}$



Let foot of the perpendicular from the point $A(2, 3, -8)$ to the given line be $B(-2\lambda + 4, 6\lambda, -3\lambda + 1)$

D-ratios of AB is: $-2\lambda + 2, 6\lambda - 3, -3\lambda + 9$

As AB is perpendicular to the given line: $-2(-2\lambda + 2) + 6(6\lambda - 3) - 3(-3\lambda + 9) = 0$

$$\Rightarrow \lambda = 1$$

\therefore Foot of the perpendicular is: $B(2, 6, -2)$

Perpendicular distance = $AB = 3\sqrt{5}$

$\frac{1}{2}$

$\frac{1}{2}$

1

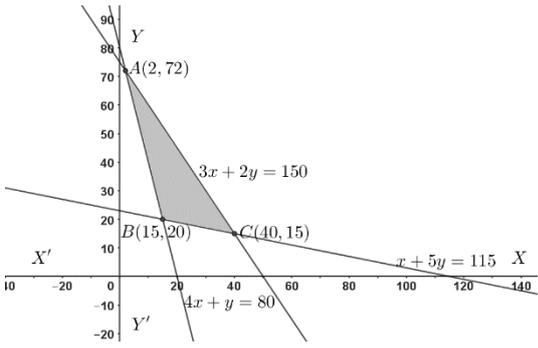
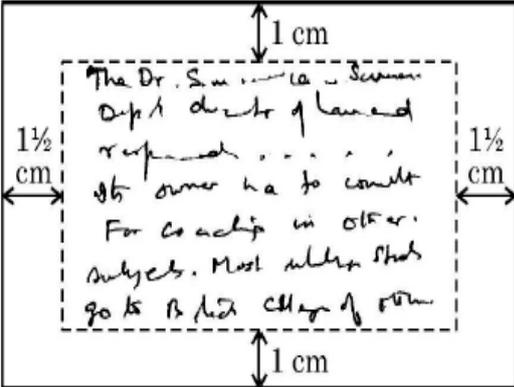
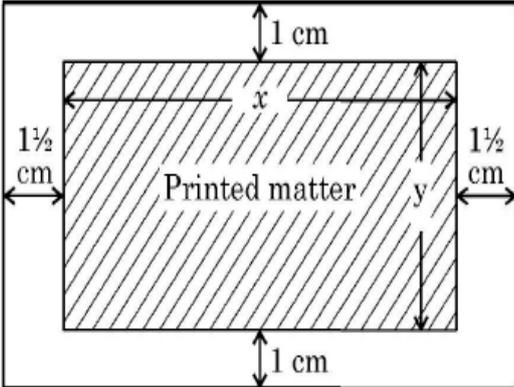
1

1

1

	<p style="text-align: center;">Or</p> <p>(b) Equation of L_1 : $\vec{r} = 2\hat{i} - \hat{j} + \hat{k} + \lambda(\hat{i} + \hat{j} + 3\hat{k})$</p> <p>Equation of L_2 : $\vec{r} = (\hat{i} + \hat{j} - 2\hat{k}) + \mu(2\hat{j} - \hat{k})$</p> <p>Taking</p> $\left. \begin{aligned} \vec{a}_1 &= 2\hat{i} - \hat{j} + \hat{k}, \vec{b}_1 = \hat{i} + \hat{j} + 3\hat{k} \\ \vec{a}_2 &= \hat{i} + \hat{j} - 2\hat{k}, \vec{b}_2 = 2\hat{j} - \hat{k} \\ \vec{a}_2 - \vec{a}_1 &= -\hat{i} + 2\hat{j} - 3\hat{k}, \vec{b}_1 \times \vec{b}_2 = -7\hat{i} + \hat{j} + 2\hat{k} \end{aligned} \right\}$ <p>Shortest Distance = $\frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{ \vec{b}_1 \times \vec{b}_2 } = \frac{1}{\sqrt{6}}$</p>	<p style="text-align: center;">1</p> <p style="text-align: center;">$\frac{1}{2}$</p> <p style="text-align: center;">$\frac{1}{2}$</p> <p style="text-align: center;">$\frac{1}{2} + 1$</p> <p style="text-align: center;">$1\frac{1}{2}$</p>
34.	<p>(a) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations :</p> $\begin{aligned} x + 2y - 3z &= 1 \\ 2x - 3z &= 2 \\ x + 2y &= 3 \end{aligned}$ <p style="text-align: center;">OR</p> <p>(b) Find the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$ $\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ and hence solve the system of linear equations :</p> $\begin{aligned} x + 2y - 3z &= -4 \\ 2x + 3y + 2z &= 2 \\ 3x - 3y - 4z &= 11 \end{aligned}$	
Ans	<p>(a) $A = 1(6) - 2(3) - 3(4) = -12 \neq 0, \therefore A^{-1}$ exist</p> $\text{adj}A = \begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}$ $\therefore A^{-1} = -\frac{1}{12} \begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}$	<p style="text-align: center;">1</p> <p style="text-align: center;">2</p> <p style="text-align: center;">$\frac{1}{2}$</p>

	<p>The given system of equations can be written as $AX = B$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$</p> $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{12} \begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1/2 \\ 2/3 \end{bmatrix}$ <p>\therefore The solution of the given system of equations is: $x = 2, y = \frac{1}{2}, z = \frac{2}{3}$</p> <p style="text-align: center;">Or</p> <p>(b) $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} = \begin{bmatrix} 67 & 0 & 0 \\ 0 & 67 & 0 \\ 0 & 0 & 67 \end{bmatrix}$</p> $\Rightarrow \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}^{-1} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ <p>Solution of the system of equations is given by:</p> $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}^{-1} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix},$ <p>$\therefore x = 3, y = -2, z = 1$</p>	<p style="text-align: center;">$1\frac{1}{2}$</p> <p style="text-align: center;">$2\frac{1}{2}$</p> <p style="text-align: center;">1</p> <p style="text-align: center;">$1\frac{1}{2}$</p>
35.	<p>Solve the following L.P.P. graphically :</p> <p>Minimise $Z = 6x + 3y$</p> <p>Subject to constraints</p> $4x + y \geq 80;$ $x + 5y \geq 115;$ $3x + 2y \leq 150$ $x, y \geq 0$	

Ans	<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="text-align: center;">  </div> <div style="text-align: right;"> <p>Correct Graph</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Corner Points</th> <th>Value of $Z = 6x + 3y$</th> </tr> </thead> <tbody> <tr> <td>A(2,72)</td> <td>Z = 228</td> </tr> <tr> <td>B(15,20)</td> <td>Z = 150</td> </tr> <tr> <td>C(40,15)</td> <td>Z = 285</td> </tr> </tbody> </table> <p style="text-align: center;">Min(Z) = 150 at x = 15, y = 20</p> </div> </div>	Corner Points	Value of $Z = 6x + 3y$	A(2,72)	Z = 228	B(15,20)	Z = 150	C(40,15)	Z = 285	$3\frac{1}{2}$ 1 $\frac{1}{2}$
Corner Points	Value of $Z = 6x + 3y$									
A(2,72)	Z = 228									
B(15,20)	Z = 150									
C(40,15)	Z = 285									
36.	<p>A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be $1\frac{1}{2}$ cm as shown below :</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <p>On the basis of the above information, answer the following questions :</p> <p>(i) Write the expression for the area of the visiting card in terms of x.</p> <p>(ii) Obtain the dimensions of the card of minimum area.</p>									
Ans	<p>(i) Let $A(x)$ be the area of the visiting card then,</p> $\text{As } xy = 24, A(x) = (x+3)(y+2) = 2x+3y+xy+6 = 2x + \frac{72}{x} + 30$ <p>(ii) $A'(x) = 2 - \frac{72}{x^2}$ and $A''(x) = \frac{144}{x^3}$,</p> <p style="text-align: center;">solving $A'(x) = 0 \Rightarrow x = 6$ is the critical point.</p> $A''(6) = \frac{144}{6} > 0, \therefore \text{Area of the card is minimum at } x = 6, y = 4$ <p>The dimension of the card with minimum area is Length = 9 cm , Breadth = 6 cm</p>	2 1 $\frac{1}{2}$ $\frac{1}{2}$								

37.	<p>A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.</p> <p>Based on the above information, answer the following questions :</p> <p>(i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time.</p> <p>Find $P(E_1)$, $P(E_2)$.</p> <p>(ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A E_1)$ and $P(A E_2)$.</p> <p>(iii) Find the probability of customer paying second month's bill in time.</p> <p style="text-align: center;">OR</p> <p>(iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.</p>	
Ans	<p>(i) $P(E_1) = \frac{7}{10} = 0.7$, $P(E_2) = \frac{3}{10} = 0.3$</p> <p>(ii) $P(A E_1) = 0.8$, $P(A E_2) = 0.4$</p> <p>(iii) $P(A) = P(E_1) \cdot P(A E_1) + P(E_2) \cdot P(A E_2) = 0.7 \times 0.8 + 0.3 \times 0.4 = 0.68$ or $\frac{17}{25}$</p>	<p>1</p> <p>1</p> <p>2</p>

Or

$$(iii) P(A) = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)} = \frac{14}{17}$$

2

38. (a) Students of a school are taken to a railway museum to learn about railways heritage and its history.



An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$$

On the basis of the above information, answer the following questions :

- (i) Find whether the relation R is symmetric or not.
- (ii) Find whether the relation R is transitive or not.
- (iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

OR

- (b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.

Ans	(a) (i) Let $(l_1, l_2) \in R \Rightarrow l_1 \parallel l_2 \Rightarrow l_2 \parallel l_1 \Rightarrow (l_2, l_1) \in R, \therefore R$ is a symmetric relation	1
	(ii) Let $(l_1, l_2), (l_2, l_3) \in R \Rightarrow l_1 \parallel l_2, l_2 \parallel l_3 \Rightarrow l_1 \parallel l_3 \Rightarrow (l_1, l_3) \in R, \therefore R$ is a transitive relation	1
	(iii) The set is $\{l : l \text{ is a line of type } y = 3x + c, c \in R\}$	2
	Or	
	(b) Let $(l_1, l_2) \in R \Rightarrow l_1 \perp l_2 \Rightarrow l_2 \perp l_1 \Rightarrow (l_2, l_1) \in R, \therefore R$ is a symmetric relation Let $(l_1, l_2), (l_2, l_3) \in R \Rightarrow l_1 \perp l_2, l_2 \perp l_3 \Rightarrow l_1 \parallel l_3 \Rightarrow (l_1, l_3) \notin R, \therefore R$ is not a transitive relation <u>** Due to printing error Part (a) or Part(b), both parts be taken as independent questions of 4 marks each</u>	2