

Series PQ2RS/2

Set – 2



प्रश्न-पत्र कोड
Q.P. Code

65/2/2

अनुक्रमांक

Roll No.

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परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains 23 printed pages.
- Please check that this question paper contains 38 questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please write down the serial number of the question in the answer-book before attempting it.
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.



गणित
MATHEMATICS



निर्धारित समय : 3 घण्टे

अधिकतम अंक : 80

Time allowed : 3 hours

Maximum Marks : 80

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

- (i) इस प्रश्न-पत्र में 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
- (ii) यह प्रश्न-पत्र पाँच खण्डों में विभाजित है – क, ख, ग, घ एवं ङ।
- (iii) खण्ड क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित 1 अंक के प्रश्न हैं।
- (iv) खण्ड ख में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के 2 अंकों के प्रश्न हैं।
- (v) खण्ड ग में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के 3 अंकों के प्रश्न हैं।
- (vi) खण्ड घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के 5 अंकों के प्रश्न हैं।
- (vii) खण्ड ङ में प्रश्न संख्या 36 से 38 प्रकरण अध्ययन आधारित 4 अंकों के प्रश्न हैं।
- (viii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
- (ix) कैल्कुलेटर का उपयोग वर्जित है।

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है।

1. अवकल समीकरण $\frac{dy}{dx} - y = 1$, दिया गया है कि $y(0) = 1$ है, के हलों की संख्या है :
(A) 0 (B) 1
(C) 2 (D) अपरिमित रूप से अनेक



2. किन्हीं दो सदिशों \vec{a} और \vec{b} के लिए, निम्नलिखित कथनों में से कौन-सा सदैव सही है ?

(A) $\vec{a} \cdot \vec{b} \geq |\vec{a}| |\vec{b}|$ (B) $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}|$

(C) $\vec{a} \cdot \vec{b} \leq |\vec{a}| |\vec{b}|$ (D) $\vec{a} \cdot \vec{b} < |\vec{a}| |\vec{b}|$

3. बिन्दु (0, 1, 2) से x-अक्ष पर डाले गए लम्ब के पाद के निर्देशांक हैं :

(A) (1, 0, 0) (B) (2, 0, 0)

(C) $(\sqrt{5}, 0, 0)$ (D) (0, 0, 0)

4. एक रैखिक प्रोग्रामन समस्या के सभी व्यवरोधों द्वारा नियत उभयनिष्ठ क्षेत्र कहलाता है :

(A) एक अपरिबद्ध क्षेत्र (B) एक इष्टतम क्षेत्र

(C) एक परिबद्ध क्षेत्र (D) एक सुसंगत क्षेत्र

5. माना किसी प्रयोग में E किसी प्रतिदर्श समष्टि S की एक घटना है, तो $P(S|E)$ बराबर है :

(A) $P(S \cap E)$ (B) $P(E)$

(C) 1 (D) 0

6. कोटि 3 के सभी अदिश आव्यूहों, जिनकी प्रत्येक प्रविष्टि -1, 0 या 1 है, की संख्या है :

(A) 1 (B) 3

(C) 2 (D) 3^9

7. $x = 1$ पर $\frac{d}{dx} [\cos (\log x + e^x)]$ बराबर है :

(A) $-\sin e$ (B) $\sin e$

(C) $-(1 + e) \sin e$ (D) $(1 + e) \sin e$

8. अवकल समीकरण $(y'')^2 + (y')^3 = x \sin (y')$ की घात :

(A) 1 है (B) 2 है

(C) 3 है (D) परिभाषित नहीं है



2. For any two vectors \vec{a} and \vec{b} , which of the following statements is always true ?
- (A) $\vec{a} \cdot \vec{b} \geq |\vec{a}| |\vec{b}|$ (B) $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}|$
(C) $\vec{a} \cdot \vec{b} \leq |\vec{a}| |\vec{b}|$ (D) $\vec{a} \cdot \vec{b} < |\vec{a}| |\vec{b}|$
3. The coordinates of the foot of the perpendicular drawn from the point (0, 1, 2) on the x-axis are given by :
- (A) (1, 0, 0) (B) (2, 0, 0)
(C) $(\sqrt{5}, 0, 0)$ (D) (0, 0, 0)
4. The common region determined by all the constraints of a linear programming problem is called :
- (A) an unbounded region (B) an optimal region
(C) a bounded region (D) a feasible region
5. Let E be an event of a sample space S of an experiment, then $P(S|E) =$
- (A) $P(S \cap E)$ (B) $P(E)$
(C) 1 (D) 0
6. The number of all scalar matrices of order 3, with each entry – 1, 0 or 1, is :
- (A) 1 (B) 3
(C) 2 (D) 3^9
7. $\frac{d}{dx} [\cos (\log x + e^x)]$ at $x = 1$ is :
- (A) $-\sin e$ (B) $\sin e$
(C) $-(1 + e) \sin e$ (D) $(1 + e) \sin e$
8. The degree of the differential equation $(y'')^2 + (y')^3 = x \sin (y')$ is :
- (A) 1 (B) 2
(C) 3 (D) not defined



9. मात्रक सदिश, जो कि सदिशों $\hat{i} + \hat{k}$ और $\hat{i} - \hat{k}$, दोनों पर लंब है, है :

(A) $2\hat{j}$

(B) \hat{j}

(C) $\frac{\hat{i} - \hat{k}}{\sqrt{2}}$

(D) $\frac{\hat{i} + \hat{k}}{\sqrt{2}}$

10. रेखा $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ के समांतर एक सदिश के दिक्-अनुपात हैं :

(A) 2, -1, 6

(B) 2, 1, 6

(C) 2, 1, 3

(D) 2, -1, 3

11. यदि $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ तथा $[F(x)]^2 = F(kx)$ है, तो k का मान है :

(A) 1

(B) 2

(C) 0

(D) -2

12. यदि एक रेखा x -अक्ष की धनात्मक दिशा के साथ 30° का कोण, y -अक्ष की धनात्मक दिशा के साथ 120° का कोण बनाती है, तो यह रेखा z -अक्ष की धनात्मक दिशा के साथ जो कोण बनाती है, वह है :

(A) 90°

(B) 120°

(C) 60°

(D) 0°

13. यदि एक 3×3 के अदिश आव्यूह के सभी अवयवों का योगफल 9 है, तो इसके सभी अवयवों का गुणनफल होगा :

(A) 0

(B) 9

(C) 27

(D) 729



9. The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is :
- (A) $2\hat{j}$ (B) \hat{j}
(C) $\frac{\hat{i} - \hat{k}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{k}}{\sqrt{2}}$
10. Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are :
- (A) 2, -1, 6 (B) 2, 1, 6
(C) 2, 1, 3 (D) 2, -1, 3
11. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is :
- (A) 1 (B) 2
(C) 0 (D) -2
12. If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is :
- (A) 90° (B) 120°
(C) 60° (D) 0°
13. If the sum of all the elements of a 3×3 scalar matrix is 9, then the product of all its elements is :
- (A) 0 (B) 9
(C) 27 (D) 729

14. समुच्चय A में परिभाषित एक तुल्यता संबंध R से बने तुल्यता वर्गों A_i ($i = 1, 2, \dots, n$) के लिए निम्नलिखित में से कौन-सा कथन सही **नहीं** है ?

(A) $\bigcup_{i=1}^n A_i = A$

(B) $A_i \cap A_j \neq \phi, i \neq j$

(C) $x \in A_i$ और $x \in A_j \Rightarrow A_i = A_j$

(D) सभी i के लिए, A_i के सभी अवयव एक-दूसरे के साथ संबंधित होते हैं

15. यदि $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$ है, तो k का मान है :

(A) 0 (B) 1

(C) 2 (D) 4

16. $f(x) = \begin{cases} |x|+3, & \text{यदि } x \leq -3 \\ -2x, & \text{यदि } -3 < x < 3 \\ 6x+2, & \text{यदि } x \geq 3 \end{cases}$

द्वारा परिभाषित फलन के असांतत्यता के बिंदुओं की संख्या है :

(A) 0 (B) 1

(C) 2 (D) अनन्त

17. फलन $f(x) = x^3 - 3x^2 + 12x - 18$:

(A) R पर निरंतर हासमान है

(B) R पर निरंतर वर्धमान है

(C) R पर न तो निरंतर वर्धमान है और न ही निरंतर हासमान है

(D) $(-\infty, 0)$ पर निरंतर हासमान है



14. Which of the following statements is **not** true about equivalence classes A_i ($i = 1, 2, \dots, n$) formed by an equivalence relation R defined on a set A ?

(A) $\bigcup_{i=1}^n A_i = A$

(B) $A_i \cap A_j \neq \phi, i \neq j$

(C) $x \in A_i$ and $x \in A_j \Rightarrow A_i = A_j$

(D) All elements of A_i are related to each other, for all i

15. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is :

(A) 0 (B) 1

(C) 2 (D) 4

16. The number of points of discontinuity of $f(x) = \begin{cases} |x|+3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x+2, & \text{if } x \geq 3 \end{cases}$ is :

(A) 0 (B) 1

(C) 2 (D) infinite

17. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is :

(A) strictly decreasing on \mathbb{R}

(B) strictly increasing on \mathbb{R}

(C) neither strictly increasing nor strictly decreasing on \mathbb{R}

(D) strictly decreasing on $(-\infty, 0)$

18. यदि $\int_0^2 2 e^{2x} dx = \int_0^a e^x dx$ है, तो 'a' का मान है :

- (A) 1 (B) 2
(C) 4 (D) $\frac{1}{2}$

प्रश्न संख्या 19 और 20 अभिकथन एवं तर्क आधारित प्रश्न हैं। दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए।

- (A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है।
(B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या नहीं करता है।
(C) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है।
(D) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है।

19. अभिकथन (A) : दो शून्येतर सदिशों \vec{a} और \vec{b} के लिए, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.

तर्क (R) : दो शून्येतर सदिशों \vec{a} और \vec{b} के लिए, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

20. अभिकथन (A) : किसी एक सममित आव्यूह A के लिए, $B'AB$ एक विषम सममित आव्यूह होता है।

तर्क (R) : एक वर्ग आव्यूह P विषम सममित आव्यूह कहलाएगा, यदि $P' = -P$.



18. If $\int_0^2 2 e^{2x} dx = \int_0^a e^x dx$, the value of 'a' is :

- (A) 1 (B) 2
(C) 4 (D) $\frac{1}{2}$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A) : For two non-zero vectors \vec{a} and \vec{b} , $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.

Reason (R) : For two non-zero vectors \vec{a} and \vec{b} , $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

20. Assertion (A) : For any symmetric matrix A, $B'AB$ is a skew-symmetric matrix.

Reason (R) : A square matrix P is skew-symmetric if $P' = -P$.

खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।

21. यदि फलन $f(x) = x + \frac{1}{x}$ ($x \neq 0$) के स्थानीय उच्चतम और स्थानीय निम्नतम मान, क्रमशः M और m द्वारा प्रदत्त हैं, तो $(M - m)$ का मान ज्ञात कीजिए।

22. मान ज्ञात कीजिए :

$$\int_0^{a^3} \frac{x^2}{x^6 + a^6} dx$$

23. दर्शाइए कि $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ अपने प्रांत में निरंतर वर्धमान है।

24. (क) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$ का मान ज्ञात कीजिए।

अथवा

(ख) फलन $f(x) = \sin^{-1}(x^2 - 4)$ का प्रांत ज्ञात कीजिए। इसका परिसर भी ज्ञात कीजिए।

25. (क) $f(x) = |\cos x|$ की $x = \frac{\pi}{2}$ पर अवकलनीयता की जाँच कीजिए।

अथवा

(ख) यदि $y = A \sin 2x + B \cos 2x$ और $\frac{d^2y}{dx^2} - ky = 0$ है, तो k का मान ज्ञात कीजिए।

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}$ ($x \neq 0$) respectively, find the value of $(M - m)$.

22. Evaluate :

$$\int_0^{a^3} \frac{x^2}{x^6 + a^6} dx$$

23. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

24. (a) Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

OR

(b) Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.

25. (a) Check the differentiability of $f(x) = |\cos x|$ at $x = \frac{\pi}{2}$.

OR

(b) If $y = A \sin 2x + B \cos 2x$ and $\frac{d^2y}{dx^2} - ky = 0$, find the value of k .

खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।

26. (क) अवकल समीकरण $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$ का विशिष्ट हल ज्ञात कीजिए; दिया गया है $y = 2$, जब $x = 1$ है।

अथवा

- (ख) अवकल समीकरण $y dx = (x + 2y^2) dy$ का व्यापक हल ज्ञात कीजिए।

27. यदि सदिश \vec{a} , \vec{b} और $2\vec{a} + 3\vec{b}$ मात्रक सदिश हों, तो \vec{a} और \vec{b} के बीच का कोण ज्ञात कीजिए।

28. पासों के एक जोड़े को एक साथ उछाला जाता है। यदि X, पासों के ऊपरी फलकों पर आई संख्याओं के पूर्ण-अंतर को व्यक्त करता है, तो X का प्रायिकता बंटन ज्ञात कीजिए।

29. ज्ञात कीजिए :

$$\int x^2 \cdot \sin^{-1}(x^{3/2}) dx$$

30. (क) यदि $x^{30} y^{20} = (x + y)^{50}$ है, तो सिद्ध कीजिए कि $\frac{dy}{dx} = \frac{y}{x}$.

अथवा

- (ख) $\frac{dy}{dx}$ ज्ञात कीजिए, यदि $5^x + 5^y = 5^{x+y}$.

31. (क) मान ज्ञात कीजिए :

$$\int_{-2}^2 \sqrt{\frac{2-x}{2+x}} dx$$

अथवा

- (ख) ज्ञात कीजिए :

$$\int \frac{1}{x[(\log x)^2 - 3 \log x - 4]} dx$$

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$; $y = 2$, when $x = 1$.

OR

- (b) Find the general solution of the differential equation :

$$y \, dx = (x + 2y^2) \, dy$$

27. If vectors \vec{a} , \vec{b} and $2\vec{a} + 3\vec{b}$ are unit vectors, then find the angle between \vec{a} and \vec{b} .

28. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X.

29. Find :

$$\int x^2 \cdot \sin^{-1}(x^{3/2}) \, dx$$

30. (a) If $x^{30}y^{20} = (x + y)^{50}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

OR

- (b) Find $\frac{dy}{dx}$, if $5^x + 5^y = 5^{x+y}$.

31. (a) Evaluate :

$$\int_{-2}^2 \sqrt{\frac{2-x}{2+x}} \, dx$$

OR

- (b) Find :

$$\int \frac{1}{x[(\log x)^2 - 3 \log x - 4]} \, dx$$

खण्ड घ

इस खण्ड में दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं।

32. p का मान ज्ञात कीजिए जिसके लिए रेखाएँ

$$\vec{r} = \lambda \hat{i} + (2\lambda + 1)\hat{j} + (3\lambda + 2)\hat{k} \text{ और}$$

$$\vec{r} = \hat{i} - 3\mu\hat{j} + (p\mu + 7)\hat{k}$$

परस्पर लंबवत और प्रतिच्छेदी भी हों। दी गई इन रेखाओं का प्रतिच्छेद बिन्दु भी ज्ञात कीजिए।

33. (क) यदि $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ तथा $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ है, तो $(AB)^{-1}$ ज्ञात कीजिए।

$|(AB)^{-1}|$ भी ज्ञात कीजिए।

अथवा

(ख) दिया गया है कि $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$, तो A^{-1} ज्ञात कीजिए। अतः इसके प्रयोग से

निम्नलिखित समीकरण निकाय

$$x + y + z = 1$$

$$2x + 3y + 2z = 2$$

$$x + y + 2z = 4$$

का हल ज्ञात कीजिए।

34. यदि प्रथम चतुर्थांश में $y^2 = 4x$, $x = 1$ और x -अक्ष से घिरे क्षेत्र का क्षेत्रफल A_1 द्वारा प्रदत्त है और $y^2 = 4x$, $x = 4$ से घिरे क्षेत्र का क्षेत्रफल A_2 द्वारा प्रदत्त है, तो $A_1 : A_2$ ज्ञात कीजिए।

35. (क) दर्शाइए कि फलन $f : \mathbb{R} \rightarrow \mathbb{R}$ जो $f(x) = \frac{2x}{1+x^2}$ द्वारा परिभाषित है, न तो एकैकी है

और न ही आच्छादक है। साथ ही, समुच्चय A ज्ञात कीजिए जिससे दिया गया फलन $f : \mathbb{R} \rightarrow A$ एक आच्छादक फलन हो जाए।

अथवा

(ख) $\mathbb{N} \times \mathbb{N}$ (जहाँ \mathbb{N} प्राकृत संख्याओं का समुच्चय है) पर एक संबंध R निम्न प्रकार प्रदत्त है :

$$(a, b) R (c, d) \Leftrightarrow a - c = b - d$$

दर्शाइए कि R एक तुल्यता संबंध है।

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. Find the value of p for which the lines

$$\vec{r} = \lambda \hat{i} + (2\lambda + 1)\hat{j} + (3\lambda + 2)\hat{k} \text{ and}$$

$$\vec{r} = \hat{i} - 3\mu\hat{j} + (p\mu + 7)\hat{k}$$

are perpendicular to each other and also intersect. Also, find the point of intersection of the given lines.

33. (a) If $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, find $(AB)^{-1}$.

Also, find $|(AB)^{-1}|$.

OR

(b) Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$, find A^{-1} . Use it to solve the following system

of equations :

$$x + y + z = 1$$

$$2x + 3y + 2z = 2$$

$$x + y + 2z = 4$$

34. If A_1 denotes the area of region bounded by $y^2 = 4x$, $x = 1$ and x -axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.

35. (a) Show that a function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither

one-one nor onto. Further, find set A so that the given function $f : \mathbb{R} \rightarrow A$ becomes an onto function.

OR

(b) A relation R is defined on $\mathbb{N} \times \mathbb{N}$ (where \mathbb{N} is the set of natural numbers) as :

$$(a, b) R (c, d) \Leftrightarrow a - c = b - d$$

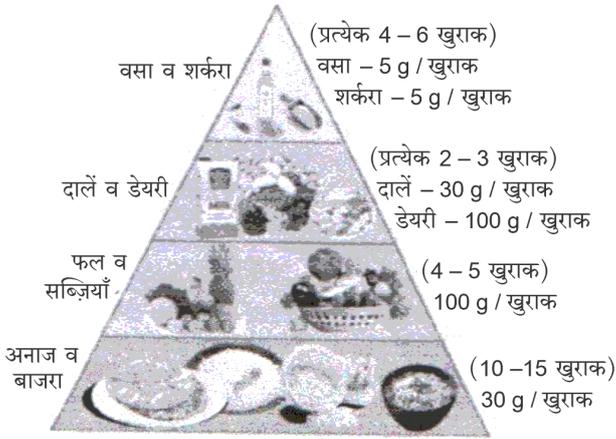
Show that R is an equivalence relation.

खण्ड ड

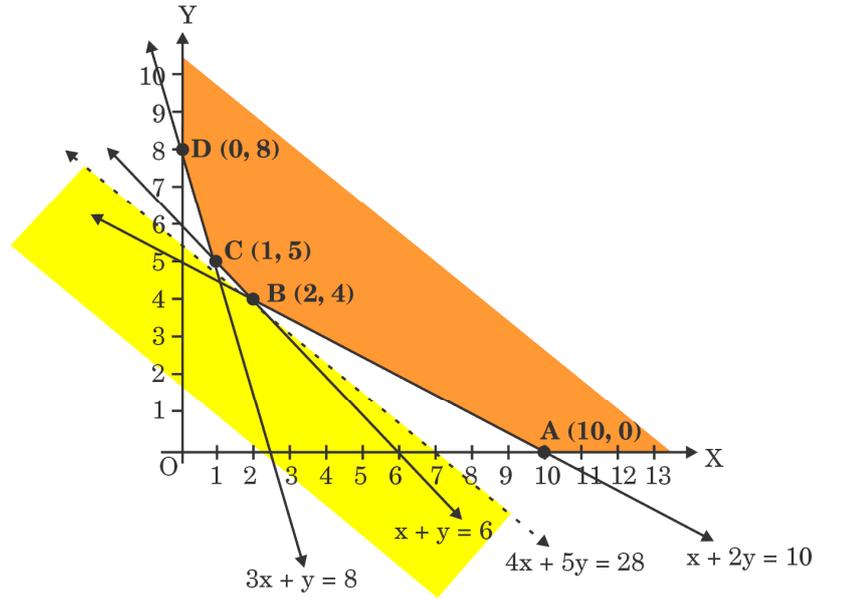
इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं।

प्रकरण अध्ययन - 1

36. सितंबर का महीना पूरे देश में राष्ट्रीय पोषण माह के रूप में मनाया जाता है। शरीर को आवश्यक उचित पोषक तत्व प्रदान करने के लिए स्वस्थ और संतुलित आहार का पालन करना महत्वपूर्ण है। संतुलित आहार हमें मानसिक रूप से भी फिट रखता है और ऊर्जा के बेहतर स्तर को बढ़ावा देता है।



आकृति-1



आकृति-2

एक आहार विशेषज्ञ दो प्रकार के खाद्य पदार्थों, फूड X (x kg) और फूड Y (y kg), जो क्रमशः ₹ 16/kg और ₹ 20/kg की दर पर उपलब्ध हैं, से युक्त आहार की लागत को कम करना चाहता है। व्यवरोधों द्वारा नियत सुसंगत क्षेत्र आकृति-2 में दिखाया गया है।

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

- आकृति-2 से उन सभी व्यवरोधों को पहचानिए और लिखिए जो दिए गए सुसंगत क्षेत्र को निर्धारित करते हैं।
- यदि उद्देश्य लागत फलन $Z = 16x + 20y$ को न्यूनतम करना हो, तो x और y का मान ज्ञात कीजिए जिस पर लागत न्यूनतम हो। यह मानते हुए कि दिए गए अपरिबद्ध क्षेत्र से न्यूनतम लागत संभव है, न्यूनतम लागत भी ज्ञात कीजिए।

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

- 36.** The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

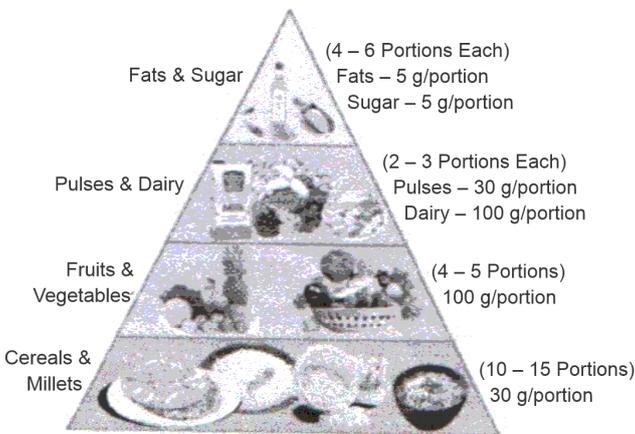


Figure-1

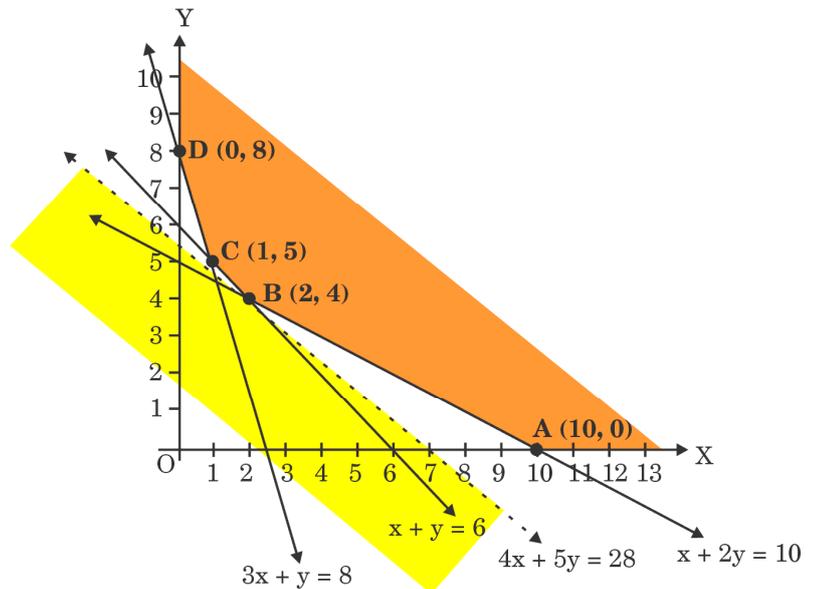


Figure-2

A dietician wishes to minimize the cost of a diet involving two types of foods, food X (x kg) and food Y (y kg) which are available at the rate of ₹ 16/kg and ₹ 20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

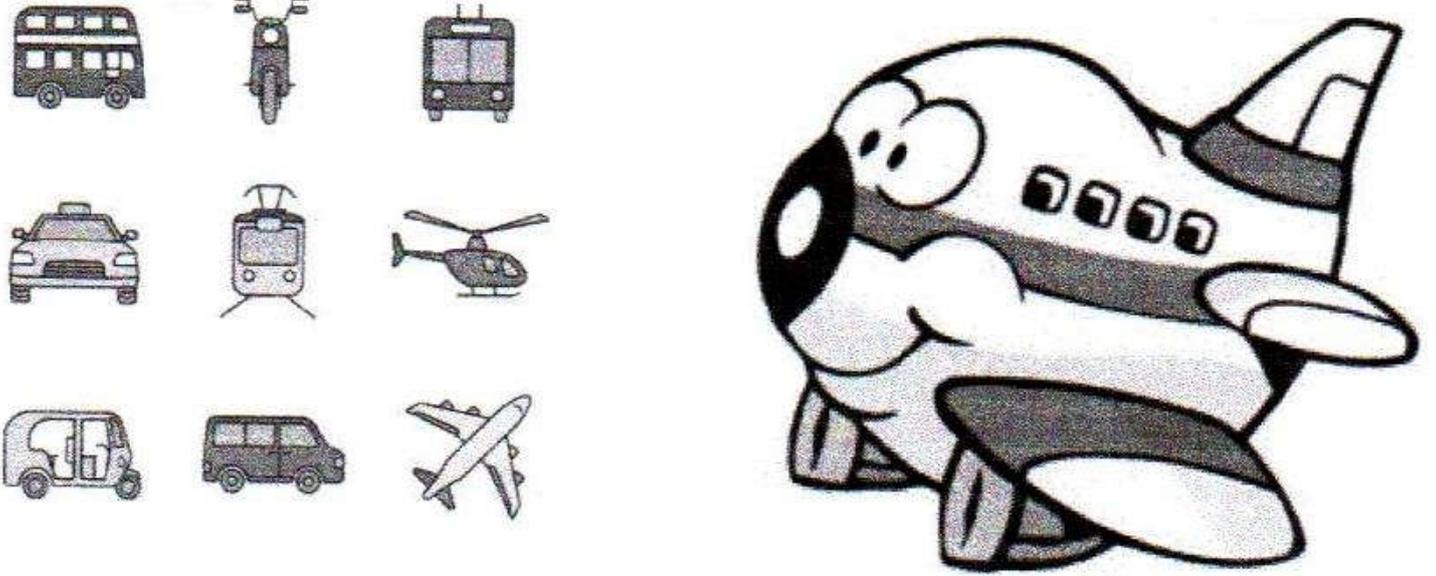
On the basis of the above information, answer the following questions :

- (i) Identify and write all the constraints which determine the given feasible region in Figure-2. 2

- (ii) If the objective is to minimize cost $Z = 16x + 20y$, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region. 2

प्रकरण अध्ययन - 2

37. जब परिवहन किए गए यात्रियों की संख्या को व्यक्तिगत चोटों और मृत्यु के योग के आधार पर मापा जाता है, तो हवाई जहाज अब तक परिवहन का सबसे सुरक्षित साधन है।



पिछले रिकॉर्ड बताते हैं कि हवाई जहाज के दुर्घटनाग्रस्त होने की प्रायिकता 0.00001% है। इसके अलावा, 95% संभावना है कि विमान दुर्घटना के बाद जीवित बचे लोग होंगे। मान लीजिए कि दुर्घटना न होने की स्थिति में सभी यात्री जीवित बच जाते हैं।

मान लीजिए E_1 घटना है कि एक विमान की दुर्घटना हुई है और E_2 वह घटना है कि कोई दुर्घटना नहीं हुई है। माना A वह घटना है जिसमें यात्री यात्रा के बाद जीवित बच जाते हैं।

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

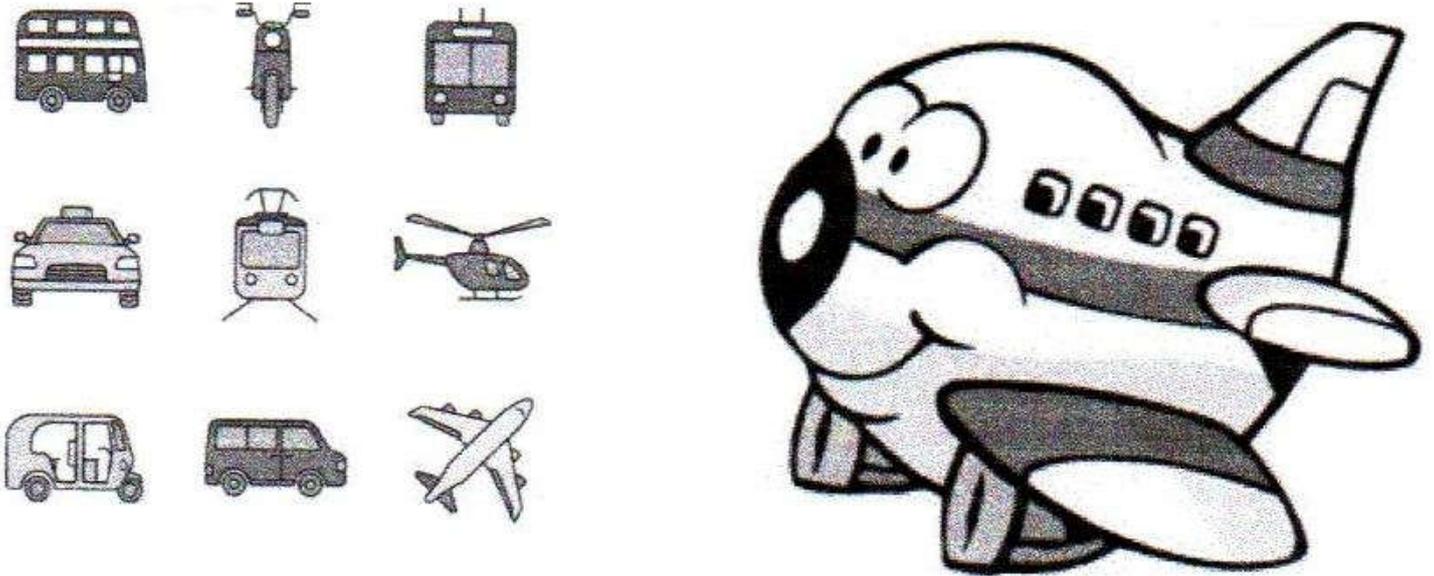
- (i) हवाई जहाज के दुर्घटनाग्रस्त न होने की प्रायिकता ज्ञात कीजिए। 1
- (ii) $P(A | E_1) + P(A | E_2)$ ज्ञात कीजिए। 1
- (iii) (क) $P(A)$ ज्ञात कीजिए। 2

अथवा

- (iii) (ख) $P(E_2 | A)$ ज्ञात कीजिए। 2

Case Study – 2

37. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.



Previous records state that the probability of an airplane crash is 0.00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

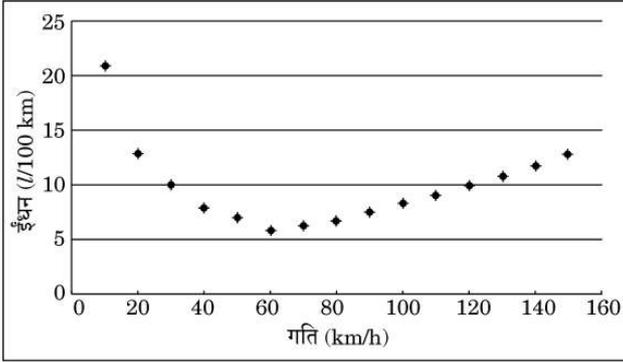
Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions :

- (i) Find the probability that the airplane will not crash. 1
 - (ii) Find $P(A | E_1) + P(A | E_2)$. 1
 - (iii) (a) Find $P(A)$. 2
- OR**
- (iii) (b) Find $P(E_2 | A)$. 2

प्रकरण अध्ययन - 3

38. ओवरस्पीडिंग से ईंधन की खपत बढ़ जाती है और टायर तथा भूमि में घर्षण और वायु प्रतिरोध के परिणामस्वरूप ईंधन की बचत कम हो जाती है। जबकि वाहन अलग-अलग गति पर इष्टतम ईंधन अर्थव्यवस्था तक पहुँचते हैं, ईंधन लाभ आमतौर पर 80 km/h से ऊपर की गति पर तेज़ी से घटता है।



कुछ बाधाओं के तहत ईंधन की खपत F (l/100 km) और गति V (km/h) के बीच संबंध

$$F = \frac{V^2}{500} - \frac{V}{4} + 14 \text{ द्वारा दिया गया है।}$$

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

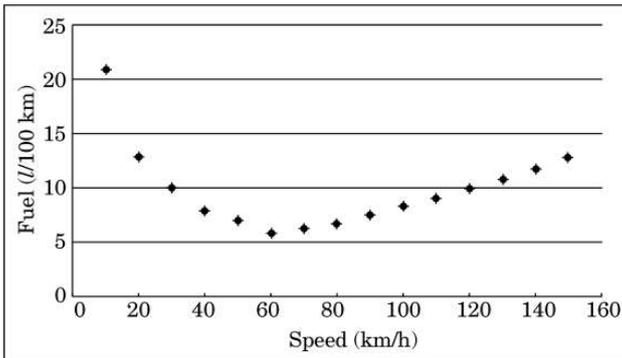
- (i) F ज्ञात कीजिए, जबकि $V = 40$ km/h. 1
- (ii) $\frac{dF}{dV}$ ज्ञात कीजिए। 1
- (iii) (क) वह गति V ज्ञात कीजिए जब ईंधन खपत F न्यूनतम होती है। 2

अथवा

- (iii) (ख) V गति से 600 km की यात्रा करने के लिए आवश्यक ईंधन की मात्रा ज्ञात कीजिए जिस पर $\frac{dF}{dV} = -0.01$ है। 2

Case Study – 3

38. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.



The relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

On the basis of the above information, answer the following questions :

- (i) Find F , when $V = 40$ km/h. 1
- (ii) Find $\frac{dF}{dV}$. 1
- (iii) (a) Find the speed V for which fuel consumption F is minimum. 2
- OR**
- (iii) (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$. 2

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only) Senior School Certificate Examination, 2024
MATHEMATICS PAPER CODE 65/2/2

General Instructions:

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its’ leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC.”
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (✓) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totalled up and written in the left- hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	<u>In Q1-Q20, if a candidate attempts the question more than once (without cancelling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note “Extra Question”.</u>

10	<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note “Extra Question”.</u>
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks _____(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- <ul style="list-style-type: none"> • Leaving answer or part thereof unassessed in an answer book. • Giving more marks for an answer than assigned to it. • Wrong totalling of marks awarded on an answer. • Wrong transfer of marks from the inside pages of the answer book to the title page. • Wrong question wise totalling on the title page. • Wrong totalling of marks of the two columns on the title page. • Wrong grand total. • Marks in words and figures not tallying/not same. • Wrong transfer of marks from the answer book to online award list. • Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) • Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totalling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the “ Guidelines for spot Evaluation ” before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totalled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each	
1.	The number of solutions of differential equation $\frac{dy}{dx} - y = 1$, given that $y(0) = 1$, is : (A) 0 (B) 1 (C) 2 (D) infinitely many	
Sol.	(B)1	1
2.	For any two vectors \vec{a} and \vec{b} , which of the following statements is always true ? (A) $\vec{a} \cdot \vec{b} \geq \vec{a} \vec{b} $ (B) $\vec{a} \cdot \vec{b} = \vec{a} \vec{b} $ (C) $\vec{a} \cdot \vec{b} \leq \vec{a} \vec{b} $ (D) $\vec{a} \cdot \vec{b} < \vec{a} \vec{b} $	
Sol.	(C) $\vec{a} \cdot \vec{b} \leq \vec{a} \vec{b} $	1
3.	The coordinates of the foot of the perpendicular drawn from the point (0, 1, 2) on the x-axis are given by : (A) (1, 0, 0) (B) (2, 0, 0) (C) ($\sqrt{5}$, 0, 0) (D) (0, 0, 0)	
Sol.	(D)(0, 0, 0)	1
4.	The common region determined by all the constraints of a linear programming problem is called : (A) an unbounded region (B) an optimal region (C) a bounded region (D) a feasible region	
Sol.	(D)a feasible region	1
5.	Let E be an event of a sample space S of an experiment, then $P(S E) =$ (A) $P(S \cap E)$ (B) $P(E)$ (C) 1 (D) 0	
Sol.	(C)1	1

6.	The number of all scalar matrices of order 3, with each entry $-1, 0$ or 1 , is : (A) 1 (B) 3 (C) 2 (D) 3^9	
Sol.	(B)3	1
7.	$\frac{d}{dx} [\cos (\log x + e^x)]$ at $x = 1$ is : (A) $-\sin e$ (B) $\sin e$ (C) $-(1 + e) \sin e$ (D) $(1 + e) \sin e$	
Sol.	(C) $-(1 + e) \sin e$	1
8.	The degree of the differential equation $(y'')^2 + (y')^3 = x \sin (y')$ is : (A) 1 (B) 2 (C) 3 (D) not defined	
Sol.	(D)not defined	1
9.	The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is : (A) $2\hat{j}$ (B) \hat{j} (C) $\frac{\hat{i} - \hat{k}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{k}}{\sqrt{2}}$	
Sol.	(B) \hat{j}	1
10.	Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are : (A) $2, -1, 6$ (B) $2, 1, 6$ (C) $2, 1, 3$ (D) $2, -1, 3$	
Sol.	(D)2, -1, 3	1
11.	If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is : (A) 1 (B) 2 (C) 0 (D) -2	
Sol.	(B)2	1

12.	<p>If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is :</p> <p>(A) 90° (B) 120° (C) 60° (D) 0°</p>	
Sol.	(A) 90°	1
13.	<p>If the sum of all the elements of a 3×3 scalar matrix is 9, then the product of all its elements is :</p> <p>(A) 0 (B) 9 (C) 27 (D) 729</p>	
Sol.	(A) 0	1
14.	<p>Which of the following statements is not true about equivalence classes A_i ($i = 1, 2, \dots, n$) formed by an equivalence relation R defined on a set A ?</p> <p>(A) $\bigcup_{i=1}^n A_i = A$ (B) $A_i \cap A_j \neq \emptyset, i \neq j$ (C) $x \in A_i$ and $x \in A_j \Rightarrow A_i = A_j$ (D) All elements of A_i are related to each other, for all i</p>	
Sol.	(B) $A_i \cap A_j \neq \emptyset, i \neq j$	1
15.	<p>If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is :</p> <p>(A) 0 (B) 1 (C) 2 (D) 4</p>	
Sol.	(D)4	1

19.	<p><i>Assertion (A) :</i> For two non-zero vectors \vec{a} and \vec{b}, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.</p> <p><i>Reason (R) :</i> For two non-zero vectors \vec{a} and \vec{b}, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.</p>	
Sol.	(C)Assertion (A) is true, but Reason (R) is false.	1
20.	<p><i>Assertion (A) :</i> For any symmetric matrix A, $B'AB$ is a skew-symmetric matrix.</p> <p><i>Reason (R) :</i> A square matrix P is skew-symmetric if $P' = -P$.</p>	
Sol.	(D)Assertion (A) is false, but Reason (R) is true.	1
SECTION B		
21	<p>If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}$ ($x \neq 0$) respectively, find the value of (M – m).</p>	
Sol.	$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2} = \frac{(x + 1)(x - 1)}{x^2}$ $f'(x) = 0 \Rightarrow x = -1, 1$ $f''(x) = \frac{2}{x^3} \Rightarrow f''(-1) = -2 < 0$ <p>$\therefore -1$ is a point of local maximum The local maximum value = $f(-1) = -2 = M$</p> $f''(1) = 2 > 0$ <p>$\therefore 1$ is point of local minimum The local minimum value = $f(1) = 2 = m$</p> $M - m = -4$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
22.	<p>Evaluate :</p> $\int_0^{a^3} \frac{x^2}{x^6 + a^6} dx$	
Sol.	<p>Put $x^3 = t \Rightarrow x^2 dx = \frac{dt}{3}$</p> <p>Given integral = $\frac{1}{3} \int_0^{a^9} \frac{dt}{t^2 + (a^3)^2}$</p>	$\frac{1}{2}$ $\frac{1}{2}$

	$= \frac{1}{3a^3} \tan^{-1} \frac{t}{a^3} \Big _0^{a^9}$ $= \frac{1}{3a^3} \tan^{-1} a^6$	$\frac{1}{2}$ $\frac{1}{2}$
23.	Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.	
Sol.	$f'(x) = e^x + e^{-x} + 1 - \frac{1}{1+x^2}$ $= e^x + \frac{1}{e^x} + \frac{x^2}{1+x^2} > 0 \text{ for all } x \in R$ <p>$\therefore f$ is strictly increasing over its domain R</p>	1 1
24.	(a) Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.	
Sol.	The given expression = $\frac{-\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4}$ $= \frac{-\pi}{12}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	OR	
	(b) Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.	
Sol.	$-1 \leq x^2 - 4 \leq 1$ $\Rightarrow 3 \leq x^2 \leq 5$ Domain = $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$ Range = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$\frac{1}{2}$ $\frac{1}{2}$ 1
25.	(a) Check the differentiability of $f(x) = \cos x $ at $x = \frac{\pi}{2}$.	
Sol.	$f(x) = \cos x = \begin{cases} \cos x & 0 \leq x \leq \frac{\pi}{2} \\ -\cos x & \frac{\pi}{2} \leq x \leq \pi \end{cases}$ $\text{LHD at } \frac{\pi}{2} = \lim_{h \rightarrow 0} \frac{f(\frac{\pi}{2}-h) - f(\frac{\pi}{2})}{-h} = \lim_{h \rightarrow 0} \frac{\cos(\frac{\pi}{2}-h) - 0}{-h} = \lim_{h \rightarrow 0} \frac{\sin h}{-h} = -1$	1

	$\text{RHD at } \frac{\pi}{2} = \lim_{h \rightarrow 0} \frac{f(\frac{\pi}{2}+h) - f(\frac{\pi}{2})}{h} = \lim_{h \rightarrow 0} \frac{-\cos(\frac{\pi}{2}+h) - 0}{h} = \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1$ <p>LHD \neq RHD</p> <p>$\therefore f$ is not differentiable at $x = \frac{\pi}{2}$</p>	$\frac{1}{2}$ $\frac{1}{2}$
	OR	
	(b) If $y = A \sin 2x + B \cos 2x$ and $\frac{d^2y}{dx^2} - ky = 0$, find the value of k .	
Sol.	$\frac{dy}{dx} = 2A \cos 2x - 2B \sin 2x$ $\Rightarrow \frac{d^2y}{dx^2} = -4A \sin 2x - 4B \cos 2x = -4y$ $\Rightarrow \frac{d^2y}{dx^2} + 4y = 0$ $\Rightarrow k = -4$	1 $\frac{1}{2}$ $\frac{1}{2}$
Section C		
26.	(a) Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$; $y = 2$, when $x = 1$.	
Sol.	<p>Given differential equation can be written as</p> $\frac{dy}{dx} = \frac{2xy + y^2}{2x^2} = \frac{y}{x} + \frac{y^2}{2x^2}$ <p>Let $y = vx \Rightarrow v + x \frac{dv}{dx}$</p> <p>The equation becomes</p> $x \frac{dv}{dx} = \frac{1}{2} v^2$ $\Rightarrow \frac{dv}{v^2} = \frac{1}{2} \times \frac{dx}{x}$ <p>Integrating both sides, we get</p> $\frac{-1}{v} = \frac{1}{2} \log x + C$ $\Rightarrow -\frac{x}{y} = \frac{1}{2} \log x + C$ <p>$x = 1, y = 2$ gives $C = -\frac{1}{2}$</p> <p>The particular solution is</p> $-\frac{x}{y} = \frac{1}{2} \log x - \frac{1}{2} \text{ or, } y = \frac{2x}{1 - \log x }$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$

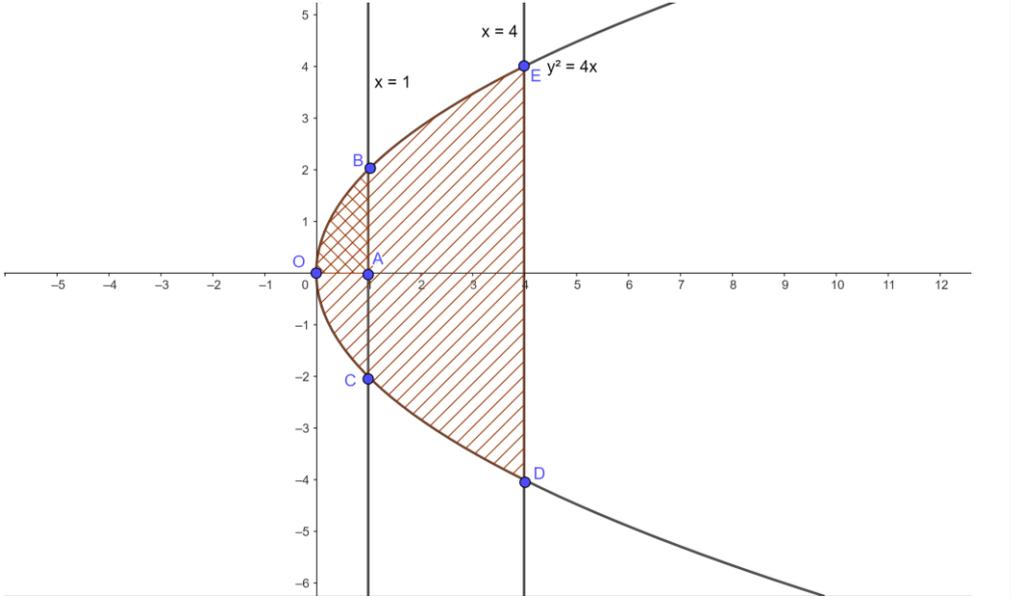
OR																
	(b) Find the general solution of the differential equation : $y \, dx = (x + 2y^2) \, dy$															
Sol.	<p>Given differential equation can be written as</p> $\frac{dx}{dy} - \frac{x}{y} = 2y$ <p>Integrating Factor = $e^{\int \frac{-1}{y} dy} = \frac{1}{y}$</p> <p>Solution is $x \frac{1}{y} = \int 2y \, dy$</p> $\Rightarrow \frac{x}{y} = 2y + C$ $\Rightarrow x = 2y^2 + Cy$	<p>1</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>														
27.	If vectors \vec{a} , \vec{b} and $2\vec{a} + 3\vec{b}$ are unit vectors, then find the angle between \vec{a} and \vec{b} .															
Sol.	$ \vec{a} = \vec{b} = 2\vec{a} + 3\vec{b} = 1$ $(2\vec{a} + 3\vec{b})^2 = 2\vec{a} + 3\vec{b} ^2$ $\Rightarrow 4 \vec{a} ^2 + 12\vec{a} \cdot \vec{b} + 9 \vec{b} ^2 = 1$ $\Rightarrow \cos\theta = -1$, where θ is angle between \vec{a} and \vec{b} Hence, $\theta = \pi$	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>														
28.	A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X.															
Sol.	<table border="1" style="width: 100%; text-align: center;"> <tr> <td>X</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>P(X)</td> <td>$\frac{6}{36} = \frac{1}{6}$</td> <td>$\frac{10}{36} = \frac{5}{18}$</td> <td>$\frac{8}{36} = \frac{2}{9}$</td> <td>$\frac{6}{36} = \frac{1}{6}$</td> <td>$\frac{4}{36} = \frac{1}{9}$</td> <td>$\frac{2}{36} = \frac{1}{18}$</td> </tr> </table>	X	0	1	2	3	4	5	P(X)	$\frac{6}{36} = \frac{1}{6}$	$\frac{10}{36} = \frac{5}{18}$	$\frac{8}{36} = \frac{2}{9}$	$\frac{6}{36} = \frac{1}{6}$	$\frac{4}{36} = \frac{1}{9}$	$\frac{2}{36} = \frac{1}{18}$	<p>$\frac{1}{2} \times 6$ $= 3$</p>
X	0	1	2	3	4	5										
P(X)	$\frac{6}{36} = \frac{1}{6}$	$\frac{10}{36} = \frac{5}{18}$	$\frac{8}{36} = \frac{2}{9}$	$\frac{6}{36} = \frac{1}{6}$	$\frac{4}{36} = \frac{1}{9}$	$\frac{2}{36} = \frac{1}{18}$										
29	<p>Find :</p> $\int x^2 \cdot \sin^{-1}(x^{3/2}) \, dx$															

Sol.	<p>Let $x^{\frac{3}{2}} = t$ $\Rightarrow \frac{3}{2} x^{\frac{1}{2}} dx = dt$</p> <p>The given integral becomes $\frac{2}{3} \int t \sin^{-1} t dt$ $= \frac{2}{3} \left[\sin^{-1} t \times \frac{t^2}{2} - \int \frac{1}{\sqrt{1-t^2}} \times \frac{t^2}{2} dt \right]$ $= \frac{1}{3} \left[\sin^{-1} t \times t^2 + \int \frac{1-t^2-1}{\sqrt{1-t^2}} dt \right]$ $= \frac{1}{3} \left[\sin^{-1} t \times t^2 + \int \sqrt{1-t^2} dt - \int \frac{1}{\sqrt{1-t^2}} dt \right]$ $= \frac{1}{3} \left[t^2 \sin^{-1} t + \frac{t}{2} \sqrt{1-t^2} + \frac{1}{2} \sin^{-1} t - \sin^{-1} t \right] + C$ $= \frac{1}{3} \left[t^2 \sin^{-1} t + \frac{t}{2} \sqrt{1-t^2} - \frac{1}{2} \sin^{-1} t \right] + C$ $= \frac{1}{3} \left[x^3 \sin^{-1} \left(x^{\frac{3}{2}} \right) + \frac{x^{\frac{3}{2}}}{2} \sqrt{1-x^3} - \frac{1}{2} \sin^{-1} \left(x^{\frac{3}{2}} \right) \right] + C$</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p>
30.	(a) If $x^{30} y^{20} = (x + y)^{50}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.	
Sol.	<p>(a) Taking log of both sides, we get $30 \log x + 20 \log y = 50 \log (x + y)$ Differentiating both sides w.r.t. x, we get $\frac{30}{x} + \frac{20}{y} \frac{dy}{dx} = \frac{50}{x+y} \left(1 + \frac{dy}{dx} \right)$ $\Rightarrow \frac{dy}{dx} \left(\frac{20x - 30y}{y(x+y)} \right) = \frac{20x - 30y}{x(x+y)}$ $\Rightarrow \frac{dy}{dx} = \frac{y}{x}$</p>	<p>1</p> <p>1</p> <p>1</p>
	OR	
	(b) Find $\frac{dy}{dx}$, if $5^x + 5^y = 5^{x+y}$.	
Sol.	<p>Differentiating both sides w.r.t. x, we get $5^x \log 5 + 5^y \log 5 \frac{dy}{dx} = 5^{x+y} \log 5 \left(1 + \frac{dy}{dx} \right)$ $\Rightarrow 5^x + 5^y \frac{dy}{dx} = (5^x + 5^y) \left(1 + \frac{dy}{dx} \right)$ $\Rightarrow 5^x + 5^y \frac{dy}{dx} = 5^x + 5^x \frac{dy}{dx} + 5^y + 5^y \frac{dy}{dx}$ $\Rightarrow \frac{dy}{dx} = -\frac{5^y}{5^x} = -5^{y-x}$</p>	<p>2</p> <p>1</p>

31.	<p>(a) Evaluate :</p> $\int_{-2}^2 \sqrt{\frac{2-x}{2+x}} dx$	
Sol.	<p>(a) $\int_{-2}^2 \sqrt{\frac{2-x}{2+x}} dx$ $= \int_{-2}^2 \frac{2-x}{\sqrt{4-x^2}} dx$ $= \int_{-2}^2 \frac{2}{\sqrt{4-x^2}} dx - \int_{-2}^2 \frac{x}{\sqrt{4-x^2}} dx$ $= 2 \int_0^2 \frac{2}{\sqrt{4-x^2}} dx - 0$ [$\frac{2}{\sqrt{4-x^2}}$ is even, $\frac{x}{\sqrt{4-x^2}}$ is odd] $= 4 \int_0^2 \frac{1}{\sqrt{4-x^2}} dx$ $= 4 \sin^{-1} \frac{x}{2} \Big _0^2$ $= 2\pi$</p>	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
OR		
	<p>(b) Find :</p> $\int \frac{1}{x[(\log x)^2 - 3 \log x - 4]} dx$	
Sol.	<p>Let $\log x = t \Rightarrow \frac{1}{x} dx = dt$ The given integral becomes $= \int \frac{1}{t^2 - 3t - 4} dt$ $= \int \frac{1}{(t - \frac{3}{2})^2 - (\frac{5}{2})^2} dx$ $= \frac{1}{5} \log \left \frac{t-4}{t+1} \right + C$ $= \frac{1}{5} \log \left \frac{\log x - 4}{\log x + 1} \right + C$</p>	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
Section D		
32.	<p>Find the value of p for which the lines $\vec{r} = \lambda \hat{i} + (2\lambda + 1)\hat{j} + (3\lambda + 2)\hat{k}$ and $\vec{r} = \hat{i} - 3\mu \hat{j} + (p\mu + 7)\hat{k}$ are perpendicular to each other and also intersect. Also, find the point of intersection of the given lines.</p>	

Sol.	<p>Given lines are</p> $\vec{r} = \hat{j} + 2\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}) \text{ and}$ $\vec{r} = \hat{i} + 7\hat{k} + \mu(-3\hat{j} + p\hat{k})$ <p>The lines are perpendicular</p> $(\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (-3\hat{j} + p\hat{k}) = 0 \Rightarrow -6 + 3p = 0 \Rightarrow p = 2$ <p>The coordinates of any point on the two lines are</p> $(\lambda, 2\lambda + 1, 3\lambda + 2) \text{ and } (1, -3\mu, 2\mu + 7)$ <p>For the point of intersection, we must have</p> $\lambda = 1, 2\lambda + 1 = -3\mu, 3\lambda + 2 = 2\mu + 7 \text{ for some } \lambda \text{ and } \mu.$ <p>Solving first two equations, we get, $\lambda = 1, \mu = -1$, which satisfies the third equation as $5 = 5$ is true.</p> <p>Hence, the lines intersect each other.</p> <p>The Point of intersection is $(1, 3, 5)$</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
33.	<p>(a) If $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, find $(AB)^{-1}$.</p> <p>Also, find $(AB)^{-1}$.</p>	
Sol	<p>We know that $(AB)^{-1} = B^{-1}A^{-1}$</p> <p>$A = 5(-1) + 4(1) = -1 \neq 0$. Hence, A^{-1} exists.</p> <p>Cofactors of the elements of A are:</p> $A_{11} = -1, A_{12} = 0, A_{13} = 1$ $A_{21} = 8, A_{22} = 1, A_{23} = -10$ $A_{31} = -12, A_{32} = -2, A_{33} = 15$ $\text{adj } A = \begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$ $A^{-1} = \frac{\text{adj } A}{ A } = \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$ $(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix} =$ $\begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$ $ (AB)^{-1} = B^{-1}A^{-1} = B^{-1} A^{-1} $ $= 1 \times \frac{1}{-1} = -1$	<p>1</p> <p>2</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p>

OR		
	<p>(b) Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$, find A^{-1}. Use it to solve the following system of equations :</p> $\begin{aligned} x + y + z &= 1 \\ 2x + 3y + 2z &= 2 \\ x + y + 2z &= 4 \end{aligned}$	
Sol.	<p>$A = 1(4) - 1(2) + 1(-1) = 1$ Cofactors of the elements of A are: $A_{11} = 4, A_{12} = -2, A_{13} = -1$ $A_{21} = -1, A_{22} = 1, A_{23} = 0$ $A_{31} = -1, A_{32} = 0, A_{33} = 1$</p> <p>$\therefore \text{adj } A = \begin{bmatrix} 4 & -1 & -1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$</p> <p>$A^{-1} = \frac{\text{adj } A}{ A } = \begin{bmatrix} 4 & -1 & -1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$</p> <p>Given system of equations can be written as $AX = B$, where</p> <p>$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$</p> <p>$X = A^{-1}B = \begin{bmatrix} 4 & -1 & -1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$</p> <p>$x = -2, y = 0, z = 3$</p>	<p>1</p> <p>2</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p>
34.	<p>If A_1 denotes the area of region bounded by $y^2 = 4x$, $x = 1$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.</p>	

<p>Sol.</p>	 <p style="text-align: right;">2</p> $A_1 = \text{Area (region OABO)} = \int_0^1 2\sqrt{x} dx = 2 \left[\frac{x^{3/2}}{3/2} \right]_0^1 = \frac{4}{3}$ $A_2 = \text{Area (region ODEO)} = 2 \int_0^4 2\sqrt{x} dx = 4 \times \frac{2}{3} [2^3] = \frac{64}{3}$ $A_1 : A_2 = \frac{4}{3} : \frac{64}{3} = 1 : 16$	<p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p>
<p>35.</p>	<p>(a) Show that a function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : \mathbb{R} \rightarrow A$ becomes an onto function.</p>	
<p>Sol.</p>	<p>Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in \mathbb{R}$ Then $\frac{2x_1}{1+x_1^2} = \frac{2x_2}{1+x_2^2}$</p> $\Rightarrow x_1 + x_1x_2^2 = x_2 + x_1^2x_2$ $\Rightarrow (x_1 - x_2) - x_1x_2(x_1 - x_2) = 0$ $\Rightarrow (x_1 - x_2)(1 - x_1x_2) = 0$ $\Rightarrow x_1 - x_2 = 0 \text{ or } 1 - x_1x_2 = 0$ $\Rightarrow x_1 = x_2 \text{ or } x_1x_2 = 1 \text{ so if } x_1x_2 = 1, x_1 \neq x_2$ <p>Hence f is not one -one</p> <p>Let $y = f(x)$ where $x \in \mathbb{R}$ Then $y = \frac{2x}{1+x^2}$. Here, for $x = 0, y = 0$ If $y \neq 0$, then $y = \frac{2x}{1+x^2}$ $\Rightarrow yx^2 - 2x + y = 0$</p>	<p style="text-align: center;">2</p>

Section E

36.

The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

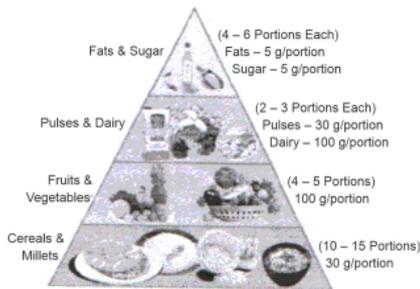


Figure-1

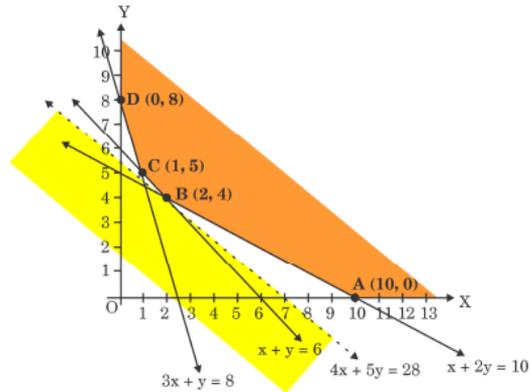


Figure-2

A dietician wishes to minimize the cost of a diet involving two types of foods, food X (x kg) and food Y (y kg) which are available at the rate of ₹ 16/kg and ₹ 20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

On the basis of the above information, answer the following questions :

- (i) Identify and write all the constraints which determine the given feasible region in Figure-2.
- (ii) If the objective is to minimize cost $Z = 16x + 20y$, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region.

Sol.

(i) Constraints are $x + 2y \geq 10$

$$x + y \geq 6$$

$$3x + y \geq 8$$

$$x \geq 0$$

$$y \geq 0$$

(ii)

Corner points	Value of $Z = 16x + 20y$
A (10, 0)	160
B (2, 4)	112
C (1, 5)	116
D (0, 8)	160

The minimum cost is ₹112

$$1\frac{1}{2}$$

$$\frac{1}{2}$$

$$1\frac{1}{2}$$

$$\frac{1}{2}$$

37.

Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.



Previous records state that the probability of an airplane crash is 0.000001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions :

- (i) Find the probability that the airplane will not crash.
 (ii) Find $P(A | E_1) + P(A | E_2)$.
 (iii) (a) Find $P(A)$.

OR

- (iii) (b) Find $P(E_2 | A)$.

Sol.

$$(i) P(E_2) = 1 - 0.00000001$$

$$= 0.99999999$$

$$(ii) P(A/E_1) + P(A/E_2) = \frac{95}{100} + 1 = \frac{195}{100}$$

1
1

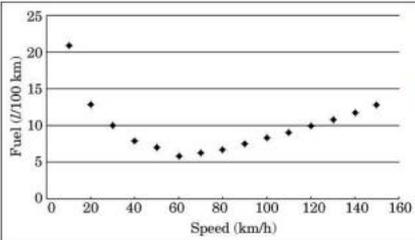
$$(iii)(a) P(A) = P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2)$$

$$= \frac{1}{10000000} \times \frac{95}{100} + \frac{99999999}{10000000} \times 1$$

$$= \frac{95+9999999900}{1000000000} = \frac{9999999995}{1000000000}$$

1

1

OR		
	(iii)(b) $P(E_2/A) = \frac{P(E_2) \times P(A/E_2)}{P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2)}$ $= \frac{\frac{9999999}{10000000}}{\frac{999999995}{1000000000}} = \frac{999999900}{999999995}$	1 1
38.	<p>Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.</p> <div style="display: flex; justify-content: space-around;">   </div> <p>The relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.</p> <p>On the basis of the above information, answer the following questions :</p> <p>(i) Find F, when $V = 40$ km/h.</p> <p>(ii) Find $\frac{dF}{dV}$.</p> <p>(iii) (a) Find the speed V for which fuel consumption F is minimum.</p> <p style="text-align: center;">OR</p> <p>(iii) (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$.</p>	
Sol.	(i) When $V = 40$ km/h, $F = 36/5$ l/100km (ii) $\frac{dF}{dV} = \frac{V}{250} - \frac{1}{4}$ (iii)(a) $\frac{dF}{dV} = 0$ $\Rightarrow V = 62.5 \text{ km/h}$	1 1 1

