

Series : YXW2Z

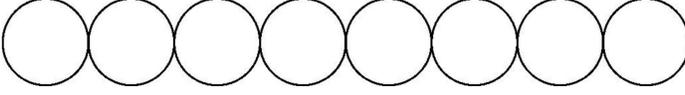


SET ~ 2



रोल नं.

Roll No.



प्रश्न-पत्र कोड
Q.P. Code

65/2/2

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.



गणित



MATHEMATICS

निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

नोट

- (I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।
- (II) कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।
- (III) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- (IV) कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।
- (V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।

NOTE

- (I) Please check that this question paper contains 23 printed pages.
- (II) Please check that this question paper contains 38 questions.
- (III) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- (IV) Please write down the serial number of the question in the answer-book at the given place before attempting it.
- (V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.



सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख्ती से पालन कीजिए :

- (i) इस प्रश्न-पत्र में कुल 38 प्रश्न हैं। सभी प्रश्न अनिवार्य हैं।
- (ii) यह प्रश्न-पत्र पाँच खण्डों में विभाजित है – खण्ड-क, ख, ग, घ एवं ङ
- (iii) खण्ड – क में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित 1 अंक के प्रश्न हैं।
- (iv) खण्ड – ख में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के 2 अंकों के प्रश्न हैं।
- (v) खण्ड – ग में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के 3 अंकों के प्रश्न हैं।
- (vi) खण्ड – घ में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के 5 अंकों के प्रश्न हैं।
- (vii) खण्ड – ङ में प्रश्न संख्या 36 से 38 प्रकरण अध्ययन आधारित 4 अंकों के प्रश्न हैं।
- (viii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड – ख के 2 प्रश्नों में, खण्ड – ग के 3 प्रश्नों में, खण्ड – घ के 2 प्रश्नों में तथा खण्ड-ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।
- (ix) कैल्कुलेटर का उपयोग वर्जित है।





General Instructions :

Read the following instructions very carefully and strictly follow them :

- (i) *This Question paper contains 38 questions. All questions are **compulsory**.*
- (ii) *Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.*
- (iii) *In **Section A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) type and Question Number **19** & **20** are Assertion-Reason based questions of **1** mark each.*
- (iv) *In **Section B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*
- (v) *In **Section C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*
- (vi) *In **Section D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*
- (vii) *In **Section E** – Question Number **36** to **38** are case study based questions, carrying **4** marks each.*
- (viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, **2** questions in Section – **D** and **2** questions in Section – **E**.*
- (ix) *Use of calculator is **NOT** allowed.*





खण्ड - क

(इस खण्ड में 20 बहुविकल्पीय प्रश्न हैं। प्रत्येक प्रश्न 1 अंक का है।)

(20 × 1 = 20)

1. x के वे मान जिनके लिए सदिशों $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$ तथा $\vec{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$ के बीच का कोण, एक अधिक कोण है, हैं :

- (A) 0 या $\frac{1}{2}$ (B) $x > \frac{1}{2}$
(C) $\left(0, \frac{1}{2}\right)$ (D) $\left[0, \frac{1}{2}\right]$

2. यदि एक रेखा x , y तथा z -अक्षों की धनात्मक दिशा से क्रमशः $\frac{3\pi}{4}$, $\frac{\pi}{3}$ तथा θ के कोण बनाती है, तो θ का मान है

- (A) केवल $\frac{-\pi}{3}$ (B) केवल $\frac{\pi}{3}$
(C) $\frac{\pi}{6}$ (D) $\pm \frac{\pi}{3}$

3. $\int \frac{dx}{\sin^2 x \cos^2 x}$ बराबर है

- (A) $\tan x + \cot x + C$ (B) $(\tan x + \cot x)^2 + C$
(C) $\tan x - \cot x + C$ (D) $(\tan x - \cot x)^2 + C$

4. माना P कोटि 3 का एक विषम सममित आव्यूह है। यदि $\det(P) = \alpha$ है, तो $(2025)^\alpha$ का मान है :

- (A) 0 (B) 1
(C) 2025 (D) $(2025)^3$

5. $\sin^{-1}\left(\cos\frac{43\pi}{5}\right)$ का मुख्य मान है

- (A) $\frac{-7\pi}{5}$ (B) $\frac{-\pi}{10}$
(C) $\frac{\pi}{10}$ (D) $\frac{3\pi}{5}$





SECTION – A

(This section comprises of **20** multiple choice questions (MCQs) of **1** mark each.)

(20 × 1 = 20)

1. The values of x for which the angle between the vectors $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$ and $\vec{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$ is obtuse, is :

(A) 0 or $\frac{1}{2}$

(B) $x > \frac{1}{2}$

(C) $\left(0, \frac{1}{2}\right)$

(D) $\left[0, \frac{1}{2}\right]$

2. If a line makes angles of $\frac{3\pi}{4}, \frac{\pi}{3}$ and θ with the positive directions of x, y and z -axis respectively, then θ is

(A) $\frac{-\pi}{3}$ only

(B) $\frac{\pi}{3}$ only

(C) $\frac{\pi}{6}$

(D) $\pm \frac{\pi}{3}$

3. $\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to

(A) $\tan x + \cot x + C$

(B) $(\tan x + \cot x)^2 + C$

(C) $\tan x - \cot x + C$

(D) $(\tan x - \cot x)^2 + C$

4. Let P be a skew-symmetric matrix of order 3. If $\det(P) = \alpha$, then $(2025)^\alpha$ is

(A) 0

(B) 1

(C) 2025

(D) $(2025)^3$

5. The principal value of $\sin^{-1}\left(\cos\frac{43\pi}{5}\right)$ is

(A) $\frac{-7\pi}{5}$

(B) $\frac{-\pi}{10}$

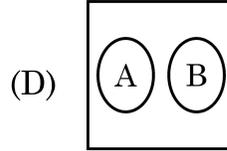
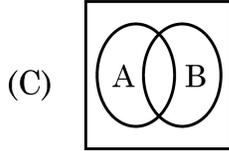
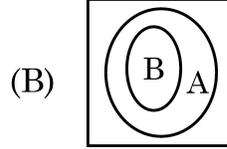
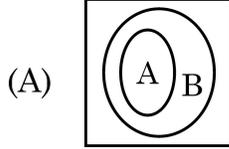
(C) $\frac{\pi}{10}$

(D) $\frac{3\pi}{5}$

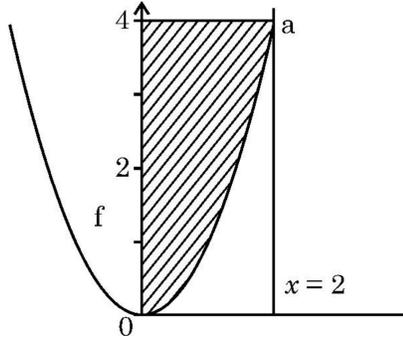




6. यदि A, संतत फलनों के समुच्चय को दर्शाता है तथा B, अवकलनीय फलनों के समुच्चय को दर्शाता है, तो निम्न में से कौन सा चित्र समुच्चय A तथा B के सही संबंध को दर्शाता है ?



7. वक्रों $y = x^2$, $0 \leq x \leq 2$ तथा y-अक्ष द्वारा दर्शाए गए छायांकित क्षेत्र का क्षेत्रफल है :



(A) $\int_0^2 x^2 dx$

(B) $\int_0^2 \sqrt{y} dy$

(C) $\int_0^4 x^2 dx$

(D) $\int_0^4 \sqrt{y} dy$

8. चार दोस्तों अभय, बीना, छाया तथा देवेश को $4AB + 3(AB + BA) - 4BA$ के सरलीकरण के लिए कहा गया, जहाँ A तथा B दोनों कोटि 2×2 के आव्यूह हैं तथा यह ज्ञात है कि $A \neq B \neq I$ तथा $A^{-1} \neq B$

उनके उत्तर इस प्रकार थे :

अभय : $6AB$

बीना : $7AB - BA$

छाया : $8AB$

देवेश : $7BA - AB$

किसका उत्तर सही है ?

(A) अभय

(B) बीना

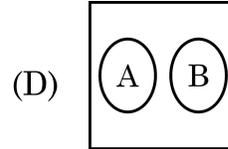
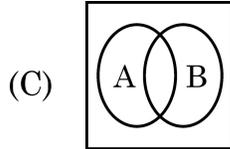
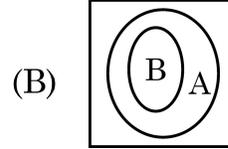
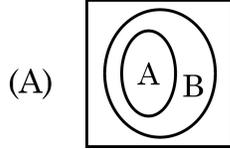
(C) छाया

(D) देवेश

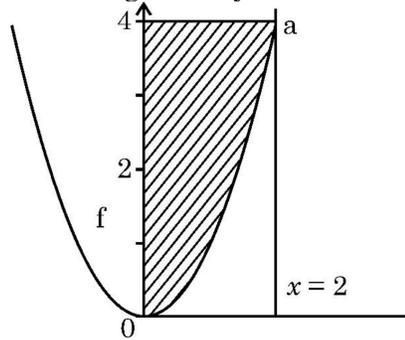




6. If A denotes the set of continuous functions and B denotes set of differentiable functions, then which of the following depicts the correct relation between set A and B ?



7. The area of the shaded region (figure) represented by the curves $y = x^2$, $0 \leq x \leq 2$ and y-axis is given by



(A) $\int_0^2 x^2 dx$

(B) $\int_0^2 \sqrt{y} dy$

(C) $\int_0^4 x^2 dx$

(D) $\int_0^4 \sqrt{y} dy$

8. Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify $4 AB + 3(AB + BA) - 4 BA$, where A and B are both matrices of order 2×2 . It is known that $A \neq B \neq I$ and $A^{-1} \neq B$.

Their answers are given as :

Abhay : $6 AB$

Bina : $7 AB - BA$

Chhaya : $8 AB$

Devesh : $7 BA - AB$

Who answered it correctly ?

(A) Abhay

(B) Bina

(C) Chhaya

(D) Devesh





9. यदि p तथा q क्रमशः अवकल समीकरण $\frac{d}{dx} \left(\frac{dy}{dx} \right)^3 = 0$, की कोटि तथा घात हैं, तो $(p - q)$

का मान है

- (A) 0 (B) 1
(C) 2 (D) 3

10. फलन $f(x) = x^2 - 4x + 6$ जिस अंतराल में वर्धमान है, वह है :

- (A) $(0, 2)$ (B) $(-\infty, 2]$
(C) $[1, 2]$ (D) $[2, \infty)$

11. निम्न प्रायिकता बंटन में p का मान है :

X	0	1	2	3
P(X)	p	p	0.3	2p

- (A) $\frac{7}{40}$ (B) $\frac{1}{10}$
(C) $\frac{9}{35}$ (D) $\frac{1}{4}$

12. यदि $\vec{PQ} \times \vec{PR} = 4\hat{i} + 8\hat{j} - 8\hat{k}$ है, तो क्षेत्रफल (ΔPQR) है :

- (A) 2 वर्ग इकाई (B) 4 वर्ग इकाई
(C) 6 वर्ग इकाई (D) 12 वर्ग इकाई

13. यदि E तथा F दो ऐसी घटनाएँ हैं कि $P(E) > 0$ तथा $P(F) \neq 1$ है, तो $P(\bar{E}/\bar{F})$ का मान है

- (A) $\frac{P(\bar{E})}{P(\bar{F})}$ (B) $1 - P(\bar{E}/F)$
(C) $1 - P(E/F)$ (D) $\frac{1 - P(E \cup F)}{P(\bar{F})}$

14. निम्न में से कौन सा आव्यूह सममित तथा विषम-सममित दोनों होते हैं ?

- (A) तत्समक आव्यूह (B) विकर्ण आव्यूह
(C) शून्य आव्यूह (D) पंक्ति आव्यूह



9. If p and q are respectively the order and degree of the differential equation

$$\frac{d}{dx} \left(\frac{dy}{dx} \right)^3 = 0, \text{ then } (p - q) \text{ is}$$

- (A) 0 (B) 1
(C) 2 (D) 3

10. The function $f(x) = x^2 - 4x + 6$ is increasing in the interval

- (A) $(0, 2)$ (B) $(-\infty, 2]$
(C) $[1, 2]$ (D) $[2, \infty)$

11. In the following probability distribution, the value of p is :

X	0	1	2	3
P(X)	p	p	0.3	$2p$

- (A) $\frac{7}{40}$ (B) $\frac{1}{10}$
(C) $\frac{9}{35}$ (D) $\frac{1}{4}$

12. If $\overrightarrow{PQ} \times \overrightarrow{PR} = 4\hat{i} + 8\hat{j} - 8\hat{k}$, then the area (ΔPQR) is

- (A) 2 sq units (B) 4 sq units
(C) 6 sq units (D) 12 sq units

13. If E and F are two events such that $P(E) > 0$ and $P(F) \neq 1$, then $P(\overline{E}/\overline{F})$ is

- (A) $\frac{P(\overline{E})}{P(\overline{F})}$ (B) $1 - P(\overline{E}/F)$
(C) $1 - P(E/F)$ (D) $\frac{1 - P(E \cup F)}{P(\overline{F})}$

14. Which of the following can be both a symmetric and skew-symmetric matrix ?

- (A) Unit Matrix (B) Diagonal Matrix
(C) Null Matrix (D) Row Matrix





15. बिंदु $(4, -3, 7)$ से होकर जाने वाली तथा सदिश $3\hat{i} + \hat{j} + 2\hat{k}$ के समांतर, रेखा का समीकरण है :
- (A) $x = 4t + 3, y = -3t + 1, z = 7t + 2$
(B) $x = 3t + 4, y = t + 3, z = 2t + 7$
(C) $x = 3t + 4, y = t - 3, z = 2t + 7$
(D) $x = 3t + 4, y = -t + 3, z = 2t + 7$
16. यदि A तथा B दोनों कोटि m के वर्ग आव्यूह हैं, तथा $A^2 - B^2 = (A - B)(A + B)$ है, तो निम्न में से कौन सा सदैव सही है ?
- (A) $A = B$ (B) $AB = BA$
(C) $A = 0$ या $B = 0$ (D) $A = I$ या $B = I$
17. रेखा $x = 1 + 5\mu, y = -5 + \mu, z = -6 - 3\mu$, निम्न में से किस बिंदु से होकर गुजरती है ?
- (A) $(1, -5, 6)$ (B) $(1, 5, 6)$
(C) $(1, -5, -6)$ (D) $(-1, -5, 6)$
18. एक फैक्टरी दो उत्पाद X तथा Y बनाती है। X तथा Y को बेचने पर प्राप्त लाभ, उद्देश्य फलन $Z = 5x + 7y$ द्वारा निरूपित है, जहाँ, x, y क्रमशः उत्पाद X तथा Y के बिकने वाले उत्पादों की संख्या है। निम्न में से कौन सा कथन सत्य है ?
- (A) उद्देश्य फलन, उत्पाद X तथा Y से प्राप्त लाभ के अंतर का अधिकतमीकरण करता है।
(B) उद्देश्य फलन, X तथा Y के उत्पादों के कुल योग को मापता है।
(C) उद्देश्य फलन, X तथा Y को बेचने से प्राप्त लाभ के योग का अधिकतमीकरण करता है।
(D) उद्देश्य फलन यह सुनिश्चित करता है कि कम्पनी उत्पाद X उत्पाद Y से अधिक बनाए।

अभिकथन एवं तर्क आधारित प्रश्न

निर्देश : प्रश्न संख्या 19 और 20 अभिकथन (A) और तर्क (R) आधारित प्रश्न हैं। दो कथन दिए गए हैं। जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए।

- (A) दोनों, अभिकथन (A) तथा तर्क (R) सही हैं। तर्क (R), अभिकथन (A) की सही व्याख्या करता है।
(B) दोनों, अभिकथन (A) तथा तर्क (R) सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या नहीं करता है।
(C) अभिकथन (A) सही है, परन्तु तर्क (R) गलत है।
(D) अभिकथन (A) गलत है जबकि तर्क (R) सही है।





15. The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point $(4, -3, 7)$ is :
- (A) $x = 4t + 3, y = -3t + 1, z = 7t + 2$
(B) $x = 3t + 4, y = t + 3, z = 2t + 7$
(C) $x = 3t + 4, y = t - 3, z = 2t + 7$
(D) $x = 3t + 4, y = -t + 3, z = 2t + 7$
16. If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct ?
- (A) $A = B$ (B) $AB = BA$
(C) $A = 0$ or $B = 0$ (D) $A = I$ or $B = I$
17. The line $x = 1 + 5\mu, y = -5 + \mu, z = -6 - 3\mu$ passes through which of the following point ?
- (A) $(1, -5, 6)$ (B) $(1, 5, 6)$
(C) $(1, -5, -6)$ (D) $(-1, -5, 6)$
18. A factory produces two products X and Y. The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct ?
- (A) The objective function maximizes the difference of the profit earned from products X and Y.
(B) The objective function measures the total production of products X and Y.
(C) The objective function maximizes the combined profit earned from selling X and Y.
(D) The objective function ensures the company produces more of product X than product Y.

ASSERTION – REASON BASED QUESTIONS

Direction : Question number 19 and 20 are Assertion (A) and Reason (R) based questions. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below :

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true but Reason (R) is false.
(D) Assertion (A) is false but Reason (R) is true.





19. अभिकथन (A) : $A = \text{diag} [3 \ 5 \ 2]$ कोटि 3×3 का एक अदिश आव्यूह है ।
तर्क (R) : यदि विकर्ण आव्यूह के सभी शून्येतर अवयव समान हों तो इसे अदिश आव्यूह कहते हैं ।
20. अभिकथन (A) : एक LPP के सुसंगत क्षेत्र का प्रत्येक बिंदु इष्टतम हल होता है ।
तर्क (R) : एक LPP का इष्टतम हल, सुसंगत क्षेत्र के एक या अधिक कोणीय बिंदु पर ही होता है ।

खण्ड – ख

(इस खण्ड में 5 अति लघु उत्तर वाले प्रश्न हैं । प्रत्येक प्रश्न के 2 अंक हैं ।)

(5 × 2 = 10)

21. 'a' के वे मान ज्ञात कीजिए जिनके लिए $f(x) = \sin x - ax + b$, R में वर्धमान है ।

22. मान ज्ञात कीजिए : $\int_0^{\pi} \frac{\sin 2px}{\sin x} dx$, $p \in \mathbb{N}$.

23. (a) यदि $x = e^{\frac{x}{y}}$ है, तो सिद्ध कीजिए कि $\frac{dy}{dx} = \frac{x-y}{x \log x}$.

अथवा

(b) यदि $f(x) = \begin{cases} 2x-3, & -3 \leq x \leq -2 \\ x+1, & -2 < x \leq 0 \end{cases}$

है, तो $x = -2$ पर $f(x)$ की अवकलनीयता की जाँच कीजिए ।

24. माना $\vec{p} = 2\hat{i} - 3\hat{j} - \hat{k}$, $\vec{q} = -3\hat{i} + 4\hat{j} + \hat{k}$ तथा $\vec{r} = \hat{i} + \hat{j} + 2\hat{k}$ है, तो \vec{r} को $\vec{r} = \lambda\vec{p} + \mu\vec{q}$ के रूप में व्यक्त कीजिए, अतः λ तथा μ के मान ज्ञात कीजिए ।



19. **Assertion (A)** : $A = \text{diag} [3 \ 5 \ 2]$ is a scalar matrix of order 3×3 .
Reason (R) : If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix.
20. **Assertion (A)** : Every point of the feasible region of a Linear Programming Problem is an optimal solution.
Reason (R) : The optimal solution for a Linear Programming Problem exists only at one or more corner point(s) of the feasible region.

SECTION – B

(This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.) (5 × 2 = 10)

21. Find the values of 'a' for which $f(x) = \sin x - ax + b$ is increasing on \mathbb{R} .

22. Evaluate : $\int_0^{\pi} \frac{\sin 2px}{\sin x} dx, p \in \mathbb{N}$.

23. (a) If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x - y}{x \log x}$.

OR

(b) If $f(x) = \begin{cases} 2x - 3, & -3 \leq x \leq -2 \\ x + 1, & -2 < x \leq 0 \end{cases}$

Check the differentiability of $f(x)$ at $x = -2$.

24. Let $\vec{p} = 2\hat{i} - 3\hat{j} - \hat{k}$, $\vec{q} = -3\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + 2\hat{k}$. Express \vec{r} in the form of $\vec{r} = \lambda\vec{p} + \mu\vec{q}$ and hence find the values of λ and μ .



25. (a) एक सदिश \vec{a} तीनों अक्षों से समान कोण बनाता है। यदि इस सदिश का परिमाण $5\sqrt{3}$ इकाई है, तो \vec{a} ज्ञात कीजिए।

अथवा

- (b) यदि दो बिंदुओं P तथा Q के स्थिति सदिश क्रमशः $\vec{\alpha}$ तथा $\vec{\beta}$ हैं, तो एक बिंदु R के स्थिति सदिश ज्ञात कीजिए जो QP के बढ़ाने पर इस प्रकार स्थित है कि $QR = \frac{3}{2} QP$.

खण्ड - ग

(इस खण्ड में 6 लघु उत्तरवाले प्रश्न हैं। प्रत्येक प्रश्न 3 अंक का है।)

(6 × 3 = 18)

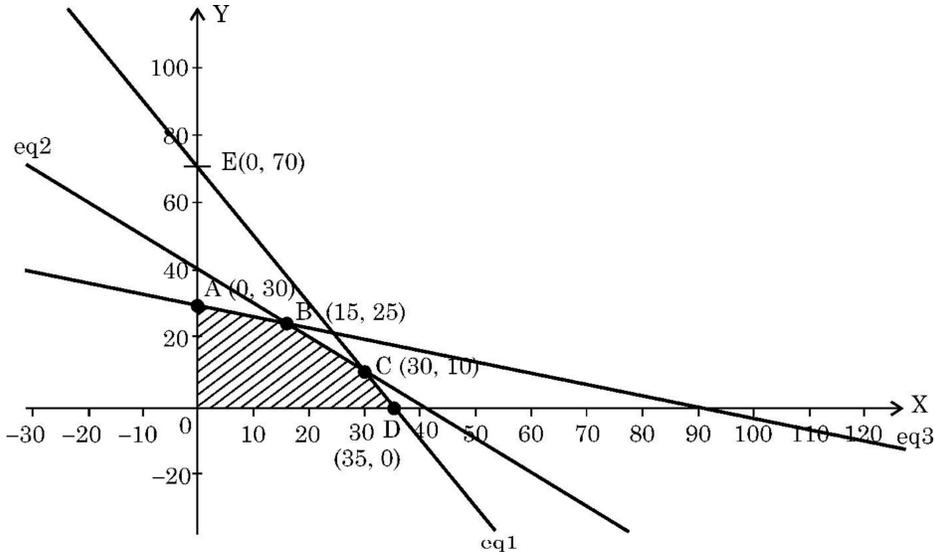
26. (a) यदि $y = \log \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2$ है, तो दर्शाइए कि $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$.

अथवा

- (b) यदि $x\sqrt{1+y} + y\sqrt{1+x} = 0$, $-1 < x < 1$, $x \neq y$, तो सिद्ध कीजिए कि $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$ ।

27. सिद्ध कीजिए कि $f : \mathbb{N} \rightarrow \mathbb{N}$, जो कि $f(x) = ax + b$ द्वारा परिभाषित है, जहाँ $a, b \in \mathbb{N}$ है, एकैकी फलन है परन्तु आच्छादक फलन नहीं।

28.



ग्राफ में एक रैखिक प्रोग्रामन समस्या का सुसंगत क्षेत्र तथा उसके कोणीय बिंदुओं के निर्देशांक दर्शाए गए हैं। इस समस्या के सभी व्यवरोधों लिखिए।



25. (a) A vector \vec{a} makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find \vec{a} .

OR

- (b) If $\vec{\alpha}$ and $\vec{\beta}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that $QR = \frac{3}{2}QP$.

SECTION - C

(This section comprises of 6 Short Answer (SA) type questions of 3 marks each.) **(6 × 3 = 18)**

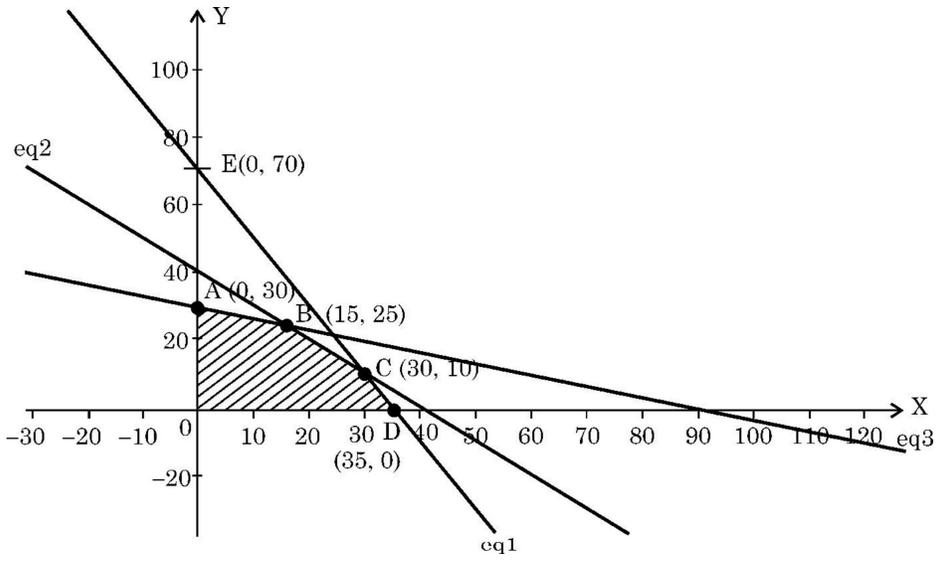
26. (a) If $y = \log \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2$, then show that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$.

OR

- (b) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, $-1 < x < 1$, $x \neq y$, then prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.

27. Prove that $f : \mathbb{N} \rightarrow \mathbb{N}$ defined as $f(x) = ax + b$ ($a, b \in \mathbb{N}$) is one-one but not onto.

28.



The feasible region along with corner points for a linear programming problem are shown in the graph. Write all the constraints for the given linear programming problem.



29. (a) अवकल समीकरण $2(y + 3) - xy \frac{dy}{dx} = 0$; का हल ज्ञात कीजिए, दिया है कि $y(1) = -2$ ।

अथवा

- (b) निम्न अवकल समीकरण को हल कीजिए :

$$(1 + x^2) \frac{dy}{dx} + 2xy = 4x^2.$$

30. (a) एक पासा जिस पर 1 से 6 तक की संख्याएँ अंकित हैं, इस प्रकार पक्षपाती है कि $P(2) = \frac{3}{10}$ है

तथा अन्य सभी संख्याओं की प्रायिकता समान है। यदि पासे को 2 बार उछाला जाए तो इस पर, जितनी बार 2 आता है, का माध्य ज्ञात कीजिए।

अथवा

- (b) दो पासों को एक साथ उछाला गया। दो घटनाएँ A तथा B निम्न प्रकार से परिभाषित की गई हैं :

$A = \{(x, y) : x + y = 9\}$, $B = \{(x, y) : x \neq 3\}$, जहाँ (x, y) प्रतिदर्श समष्टि के एक बिंदु को दर्शाता है। जाँच कीजिए कि क्या घटनाएँ A तथा B स्वतंत्र हैं या परस्पर अपवर्जी हैं।

31. f तथा g अंतराल $[a, b]$ पर संतत फलन हैं तथा यह दिया है कि $f(a - x) = f(x)$ तथा

$$g(x) + g(a - x) = a \text{ है, तो दर्शाइए कि } \int_0^a f(x) g(x) dx = \frac{a}{2} \int_0^a f(x) dx.$$



29. (a) Solve the differential equation $2(y + 3) - xy \frac{dy}{dx} = 0$; given $y(1) = -2$.

OR

(b) Solve the following differential equation :

$$(1 + x^2) \frac{dy}{dx} + 2xy = 4x^2.$$

30. (a) A die with number 1 to 6 is biased such that $P(2) = \frac{3}{10}$ and probability of other numbers is equal. Find the mean of the number of times number 2 appears on the dice, if the dice is thrown twice.

OR

(b) Two dice are thrown. Defined are the following two events A and B :

$A = \{(x, y) : x + y = 9\}$, $B = \{(x, y) : x \neq 3\}$, where (x, y) denote a point in the sample space.

Check if events A and B are independent or mutually exclusive.

31. f and g are continuous functions on interval $[a, b]$. Given that $f(a - x) = f(x)$

and $g(x) + g(a - x) = a$, show that $\int_0^a f(x) g(x) dx = \frac{a}{2} \int_0^a f(x) dx$.





खण्ड - घ

(इस खण्ड में 4 दीर्घ उत्तर वाले प्रश्न हैं। प्रत्येक प्रश्न के 5 अंक हैं।)

(4 × 5 = 20)

32. (a) निम्न रेखाओं के बीच की न्यूनतम दूरी ज्ञात कीजिए :

$$\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3} \text{ तथा}$$

$$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}.$$

अथवा

(b) बिंदु A(2, 1, 2) की रेखा $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda (\hat{i} - \hat{j} - \hat{k})$ में प्रतिबिंब A' ज्ञात कीजिए। रेखा AA' का समीकरण भी ज्ञात कीजिए तथा A से रेखा l पर डाले गए लंब का लंबपाद ज्ञात कीजिए।

33. ज्ञात कीजिए : $\int \frac{5x}{(x+1)(x^2+9)} dx.$

34. (a) दिया है कि $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ तथा $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, AB ज्ञात कीजिए, अतः

रैखिक समीकरण युग्म

$$x - y + z = 4$$

$$x - 2y - 2z = 9$$

$$2x + y + 3z = 1 \text{ को हल कीजिए।}$$

अथवा

(b) यदि $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए।

अतः निम्न रैखिक समीकरण निकाय को हल कीजिए :

$$x - 2y = 10$$

$$2x - y - z = 8$$

$$-2y + z = 7$$

35. समाकलन के प्रयोग से, रेखा $y = 5x + 2$, x -अक्ष तथा $x = -2$ और $x = 2$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए।





SECTION - D

(This section comprises of 4 Long Answer (LA) type questions of 5 marks each.) (4 × 5 = 20)

32. (a) Find the shortest distance between the lines :

$$\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3} \text{ and}$$

$$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}.$$

OR

- (b) Find the image A' of the point $A(2, 1, 2)$ in the line $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda (\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of line joining AA' . Find the foot of perpendicular from point A on the line l .

33. Find : $\int \frac{5x}{(x+1)(x^2+9)} dx$.

34. (a) Given $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB . Hence, solve

the system of linear equations :

$$x - y + z = 4$$

$$x - 2y - 2z = 9$$

$$2x + y + 3z = 1$$

OR

- (b) If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find A^{-1} .

Hence, solve the system of linear equations :

$$x - 2y = 10$$

$$2x - y - z = 8$$

$$-2y + z = 7$$

35. Using integration, find the area of the region bounded by the line $y = 5x + 2$, the x -axis and the ordinates $x = -2$ and $x = 2$.

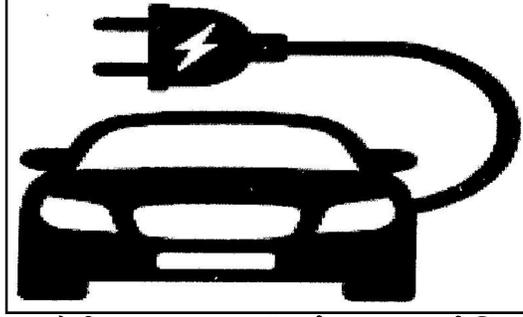




खण्ड – ड

(इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं। प्रत्येक प्रश्न के 4 अंक हैं।) (3 × 4 = 12)

36. तीन व्यक्ति अम्बर, बोन्जी तथा कामेट ऐसी कारें बना रहे हैं जो पेट्रोल पर चलती हैं तथा बैटरी पर भी चलती हैं। मार्केट में उनके उत्पाद का हिस्सा क्रमशः 60%, 30% तथा 10% है। उनके कुल उत्पाद का 20%, 10% तथा 5% क्रमशः इलेक्ट्रिक (अर्थात् बैटरी पर चलने वाली) कारें हैं। उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :



- (i) (a) क्या प्रायिकता है कि यादृच्छया चुनी गई कार एक इलेक्ट्रिक कार है ? 2
अथवा
(i) (b) क्या प्रायिकता है कि यादृच्छया चुनी गई कार एक पेट्रोल से चलने वाली कार है ? 2
(ii) यादृच्छया चुनी गई कार इलेक्ट्रिक पाई गई। क्या प्रायिकता है कि यह कामेट द्वारा बनाई गई है ? 1
(iii) यादृच्छया चुनी गई कार इलेक्ट्रिक पाई गई। क्या प्रायिकता है कि यह अम्बर या बोन्जी द्वारा बनाई गई है ? 1

37.



एक छोटा शहर नई सड़क लाईट स्थापन के पैटर्न का विश्लेषण कर रहा है। लाइटों इस तरह से स्थापित की जाती हैं कि सड़क की शुरुआत से x मीटर की दूरी पर किसी बिंदु पर प्रकाश की तीव्रता को $f(x) = e^x \sin x$ द्वारा मॉडल किया जा सकता है, जहाँ x मीटरों में है।

उपरोक्त के आधार पर निम्न के उत्तर दीजिए :

- (i) वह अंतराल ज्ञात कीजिए जहाँ $f(x)$ वर्धमान अथवा हासमान है, $x \in [0, \pi]$. 2
(ii) जाँच कीजिए कि $x \in [0, \pi]$ में प्रत्येक क्रांतिक बिंदु स्थानीय उच्चतम या स्थानीय न्यूनतम अथवा नति परिवर्तन बिंदु है। 2



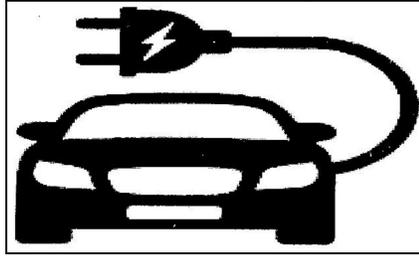
SECTION – E

(This section comprises of 3 case study based questions of 4 marks each.)

(3 × 4 = 12)

36. Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively. Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated).

Based on the above, answer the following :



- (i) (a) What is the probability that a randomly selected car is an electric car ? 2
- OR**
- (i) (b) What is the probability that a randomly selected car is a petrol car ? 2
- (ii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet ? 1
- (iii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi ? 1

37.



A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres.

Based on the above, answer the following :

- (i) Find the intervals on which the $f(x)$ is increasing or decreasing, $x \in [0, \pi]$. 2
- (ii) Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion. 2





38. एक विद्यालय में वाद-विवाद प्रतियोगिता का आयोजन किया जा रहा है, जिसमें वक्ताओं के रूप में $S = \{S_1, S_2, S_3, S_4\}$ तथा इनका निर्णय लेने के लिए जज $J = \{J_1, J_2, J_3\}$ के रूप में भाग ले रहे हैं, जहाँ प्रत्येक वक्ता के लिए एक जज निर्धारित किया जा सकता है। माना R एक समुच्चय S से J का संबंध इस प्रकार परिभाषित है : $R = \{(x, y) : \text{वक्ता } x \text{ के लिए जज } y \text{ निर्धारित किया है, } x \in S, y \in J\}$



उपरोक्त के आधार पर निम्न प्रश्नों के उत्तर दीजिए :

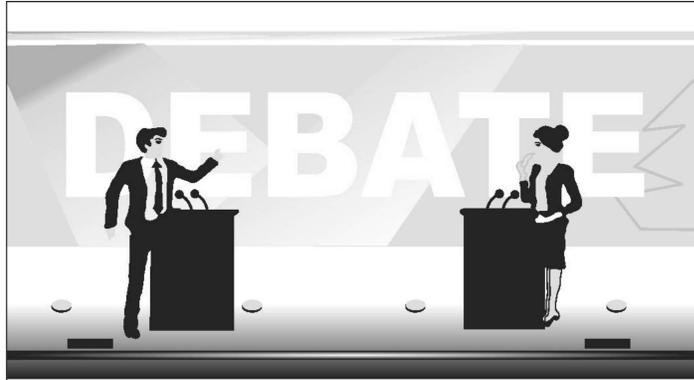
- (i) S से J तक कितने संबंध सम्भव हैं ? 1
- (ii) एक विद्यार्थी ने S से J तक का एक फलन निम्न प्रकार से परिभाषित किया :
 $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$ जाँच कीजिए कि क्या यह फलन एकैकी-
आच्छादक है ? 1
- (iii) (a) समुच्चय S से समुच्चय J में कुल कितने एकैकी फलन परिभाषित किए जा सकते हैं ? 2

अथवा

- (iii) (b) एक अन्य विद्यार्थी ने समुच्चय S में एक संबंध $R_1 = \{(S_1, S_2), (S_2, S_4)\}$ द्वारा परिभाषित किया। वह न्यूनतम क्रमित युग्म लिखिए जो R_1 में जोड़ने पर यह स्वतुल्य हो जाए परंतु सममित ना हो। 2



38. A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}$.



Based on the above, answer the following :

- (i) How many relations can be there from S to J ? 1
- (ii) A student identifies a function from S to J as $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$ Check if it is bijective. 1
- (iii) (a) How many one-one functions can be there from set S to set J ? 2

OR

- (iii) (b) Another student considers a relation $R_1 = \{(S_1, S_2), (S_2, S_4)\}$ in set S . Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric. 2



**Marking Scheme
Strictly
Confidential
(For Internal and Restricted use only)
Senior Secondary Examination, 2025
SUBJECT NAME MATHEMATICS (Q.P. CODE – 65/2/2)**

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC.”
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. This is the most common mistake which evaluators are committing.
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left- hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note “Extra Question” .

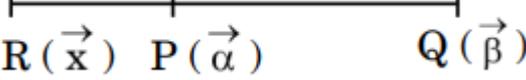
10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks_____ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - <ul style="list-style-type: none"> ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of the answer marked correct and the rest as wrong, but no marks awarded.
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the “ Guidelines for Spot Evaluation ” before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

11.	<p>In the following probability distribution, the value of p is :</p> <table border="1" data-bbox="309 248 1046 360"> <tbody> <tr> <td>X</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>P(X)</td> <td>p</td> <td>p</td> <td>0.3</td> <td>2p</td> </tr> </tbody> </table> <p>(A) $\frac{7}{40}$ (B) $\frac{1}{10}$ (C) $\frac{9}{35}$ (D) $\frac{1}{4}$</p>	X	0	1	2	3	P(X)	p	p	0.3	2p	
X	0	1	2	3								
P(X)	p	p	0.3	2p								
Ans	(A) $\frac{7}{40}$	1										
12.	<p>If $\vec{PQ} \times \vec{PR} = 4\hat{i} + 8\hat{j} - 8\hat{k}$, then the area ($\Delta PQR$) is</p> <p>(A) 2 sq units (B) 4 sq units (C) 6 sq units (D) 12 sq units</p>											
Ans	(C) 6 sq units	1										
13.	<p>If E and F are two events such that $P(E) > 0$ and $P(F) \neq 1$, then $P(\overline{E}/\overline{F})$ is</p> <p>(A) $\frac{P(\overline{E})}{P(\overline{F})}$ (B) $1 - P(\overline{E}/F)$ (C) $1 - P(E/F)$ (D) $\frac{1 - P(E \cup F)}{P(\overline{F})}$</p>											
Ans	(D) $\frac{1 - P(E \cup F)}{P(\overline{F})}$	1										
14.	<p>Which of the following can be both a symmetric and skew-symmetric matrix ?</p> <p>(A) Unit Matrix (B) Diagonal Matrix (C) Null Matrix (D) Row Matrix</p>											
Ans	(C) Null Matrix	1										

15.	<p>The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point $(4, -3, 7)$ is :</p> <p>(A) $x = 4t + 3, y = -3t + 1, z = 7t + 2$ (B) $x = 3t + 4, y = t + 3, z = 2t + 7$ (C) $x = 3t + 4, y = t - 3, z = 2t + 7$ (D) $x = 3t + 4, y = -t + 3, z = 2t + 7$</p>	
Ans	(C) $x = 3t + 4, y = t - 3, z = 2t + 7$	1
16.	<p>If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct ?</p> <p>(A) $A = B$ (B) $AB = BA$ (C) $A = 0$ or $B = 0$ (D) $A = I$ or $B = I$</p>	
Ans	(B) $AB = BA$	1
17.	<p>The line $x = 1 + 5\mu, y = -5 + \mu, z = -6 - 3\mu$ passes through which of the following point ?</p> <p>(A) $(1, -5, 6)$ (B) $(1, 5, 6)$ (C) $(1, -5, -6)$ (D) $(-1, -5, 6)$</p>	
Ans	(C) $(1, -5, -6)$	1
18.	<p>A factory produces two products X and Y. The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct ?</p> <p>(A) The objective function maximizes the difference of the profit earned from products X and Y. (B) The objective function measures the total production of products X and Y. (C) The objective function maximizes the combined profit earned from selling X and Y. (D) The objective function ensures the company produces more of product X than product Y.</p>	
Ans	(C) The objective function maximizes the combined profit earned from selling X and Y	1

22.	<p>Evaluate : $\int_0^{\pi} \frac{\sin 2px}{\sin x} dx, p \in \mathbb{N}.$</p>	
Ans	$I = \int_0^{\pi} \frac{\sin 2px}{\sin x} dx$ $= \int_0^{\pi} \frac{\sin 2p(\pi - x)}{\sin(\pi - x)} dx$ $I = \int_0^{\pi} \frac{-\sin 2px}{\sin x} dx$ <p>Adding, we get</p> $2I = 0$ $\therefore I = 0$	<p>1</p> <p>1</p>
23	<p>(a) If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x - y}{x \log x}$.</p> <p style="text-align: center;">OR</p> <p>(b) If $f(x) = \begin{cases} 2x - 3, & -3 \leq x \leq -2 \\ x + 1, & -2 < x \leq 0 \end{cases}$</p> <p>Check the differentiability of $f(x)$ at $x = -2$.</p>	
23 (a) Ans	$x = e^{\frac{x}{y}}$ $\Rightarrow \log x = \frac{x}{y}$ $\Rightarrow y \log x = x$ <p>Differentiating both sides w.r.to x, we get</p> $\frac{y}{x} + \log x \frac{dy}{dx} = 1$	<p>½</p> <p>1</p>

25 (a) Ans	<p>Let α be the angle which the vector \vec{a} makes with all the three axes.</p> <p>Then $3\cos^2\alpha = 1$</p> $\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$ <p>The unit vector along the vector $\vec{a} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$</p> $\vec{a} = 5(\hat{i} + \hat{j} + \hat{k})$ <p style="text-align: center;">OR</p>	<p>1</p> <p>½</p> <p>½</p>
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25 (b) Ans	 <p>$\frac{QR}{QP} = \frac{3}{2}$</p> <p>Hence, R divides PQ, externally, in the ratio 1:3.</p> <p>The Position vector of R = $\vec{x} = \frac{\vec{\beta} - 3\vec{\alpha}}{1-3} = \frac{3\vec{\alpha} - \vec{\beta}}{2}$</p>	<p>1</p> <p>1</p>
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SECTION-C

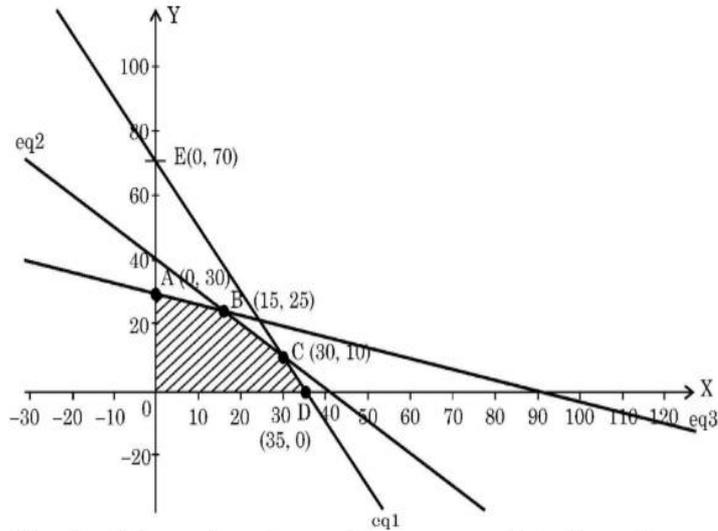
This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26	<p>(a) If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then show that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$.</p> <p style="text-align: center;">OR</p> <p>(b) If $x\sqrt{1+y} + y\sqrt{1+x} = 0, -1 < x < 1, x \neq y$, then prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.</p>	
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26(a) Ans	<p>The given function can be written as</p> $y = 2 \log(x+1) - \log x$ $\Rightarrow y_1 = \frac{2}{x+1} - \frac{1}{x} = \frac{x-1}{x(x+1)}$ $\Rightarrow (x+1)y_1 = \frac{x-1}{x} = 1 - \frac{1}{x}$ $\Rightarrow (x+1)y_2 + y_1 = \frac{1}{x^2}$	<p>1</p> <p>1</p>
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	$\Rightarrow x(x+1)^2 y_2 + x(x+1)y_1 = 1 + \frac{1}{x}$ $\Rightarrow x(x+1)^2 y_2 + x(x+1)y_1 = 1 + 1 - (x+1)y_1$ $\Rightarrow x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$ <p style="text-align: center;">OR</p>	1
26(b) Ans	$x\sqrt{1+y} + y\sqrt{1+x} = 0$ $\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$ $\Rightarrow x^2(1+y) = y^2(1+x)$ $\Rightarrow (x-y)(x+y) + xy(x-y) = 0$ $\Rightarrow (x-y)(x+y+xy) = 0$ $x \neq y \Rightarrow x+y+xy = 0$ $\Rightarrow y = \frac{-x}{1+x}$ $\Rightarrow \frac{dy}{dx} = \frac{-1}{(x+1)^2}$	 1/2 1 1/2 1
27.	Prove that $f : \mathbb{N} \rightarrow \mathbb{N}$ defined as $f(x) = ax + b$ ($a, b \in \mathbb{N}$) is one-one but not onto.	
Ans	Let $x_1, x_2 \in \mathbb{N}$ (Domain) such that $f(x_1) = f(x_2)$ $\Rightarrow ax_1 + b = ax_2 + b$ $\Rightarrow x_1 = x_2$ Therefore, f is one-one. Let $y \in \mathbb{N}$ (codomain). Then $f(x) = y$ if, $ax + b = y$ i.e., if, $x = \frac{y-b}{a}$, which may not belong to \mathbb{N} (domain) Therefore, f is not onto.	 1/2 1/2

28.



The feasible region along with corner points for a linear programming problem are shown in the graph. Write all the constraints for the given linear programming problem.

Ans

The equation of the line AB is

$$y - 30 = \frac{25-30}{15-0}(x - 0)$$

$$\Rightarrow x + 3y = 90$$

The equation of the line BC is

$$y - 10 = \frac{10-25}{30-15}(x - 30)$$

$$\Rightarrow x + y = 40$$

The equation of the line CD is

$$y - 10 = \frac{70-10}{0-30}(x - 30)$$

$$\Rightarrow 2x + y = 70$$

Hence, the constraints are

$$x + 3y \leq 90, x + y \leq 40, 2x + y \leq 70$$

$$x \geq 0, y \geq 0$$

1½

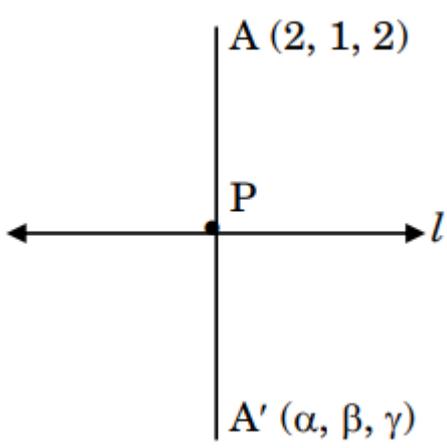
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29	<p>(a) Solve the differential equation $2(y + 3) - xy \frac{dy}{dx} = 0$; given $y(1) = -2$.</p> <p style="text-align: center;">OR</p> <p>(b) Solve the following differential equation :</p> $(1 + x^2) \frac{dy}{dx} + 2xy = 4x^2.$	
29(a) Ans	<p>Given differential equation can be written as</p> $\frac{y}{y+3} dy = \frac{2}{x} dx$ $\Rightarrow \int \left(1 - \frac{3}{y+3}\right) dy = 2 \int \frac{1}{x} dx$ $\Rightarrow y - 3 \log y + 3 = 2 \log x + C$ <p>$y = -2$, when $x = 1 \Rightarrow C = -2$</p> <p>Hence, the required particular solution is</p> $\Rightarrow y - 3 \log y + 3 = 2 \log x - 2$ <p style="text-align: center;">OR</p>	1 1½ ½
29(b) Ans	<p>Given differential equation can be written as</p> $\frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{4x^2}{1+x^2}, \text{ which is linear in } y.$ $\text{I.F.} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = 1 + x^2$ <p>The solution is given by</p> $y(1 + x^2) = \int 4x^2 dx$ $\Rightarrow y(1 + x^2) = \frac{4}{3} x^3 + C$ <p>or $y = \frac{4x^3}{3(1+x^2)} + C \frac{1}{1+x^2}$, which is the required general solution</p>	1 1 1

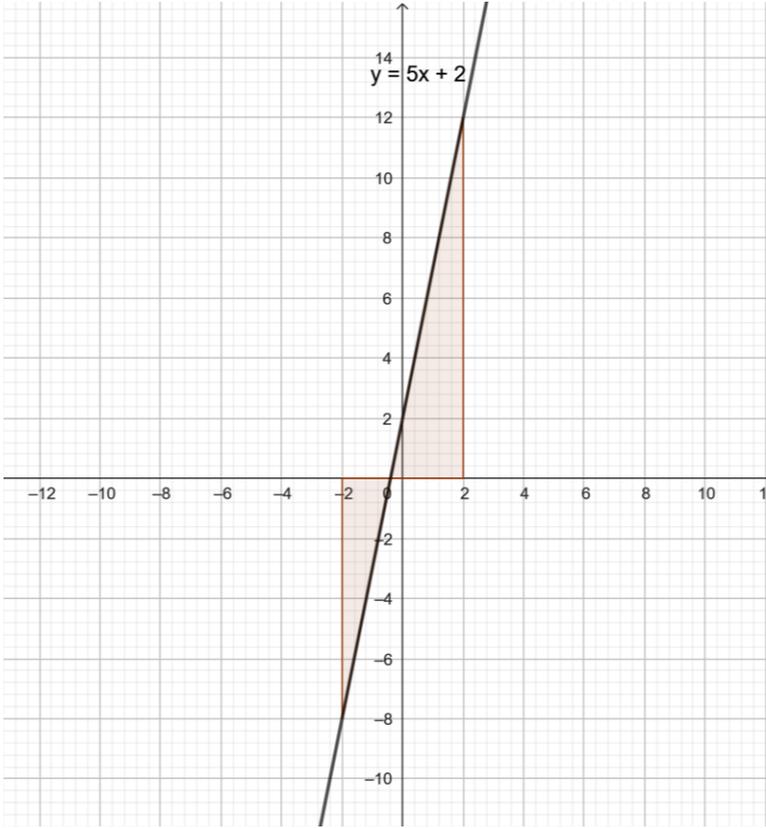
30	<p>(a) A die with number 1 to 6 is biased such that $P(2) = \frac{3}{10}$ and probability other numbers is equal. Find the mean of the number of times number appears on the dice, if the dice is thrown twice.</p> <p style="text-align: center;">OR</p> <p>(b) Two dice are thrown. Defined are the following two events A and B : $A = \{(x, y) : x + y = 9\}$, $B = \{(x, y) : x \neq 3\}$, where (x, y) denote a point in the sample space.</p> <p>Check if events A and B are independent or mutually exclusive.</p>													
30(a) Ans	<p>$P(2) = \frac{3}{10}$, $P(\text{any other number}) = 1 - \frac{3}{10} = \frac{7}{10}$</p> <p>Let X represent the Random Variable “the number of 2’s”.</p> <p>Then $X = 0, 1, 2$</p> <p>The probability distribution is</p> <table border="1" data-bbox="300 902 1217 1160"> <thead> <tr> <th>X</th> <th>P(X)</th> <th>XP(X)</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>$\frac{7}{10} \times \frac{7}{10} = \frac{49}{100}$</td> <td>0</td> </tr> <tr> <td>1</td> <td>$\frac{3}{10} \times \frac{7}{10} \times 2 = \frac{42}{100}$</td> <td>$\frac{42}{100}$</td> </tr> <tr> <td>2</td> <td>$\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$</td> <td>$\frac{18}{100}$</td> </tr> </tbody> </table> <p>Mean = $\sum XP(X) = \frac{60}{100} = 0.6$</p> <p style="text-align: center;">OR</p>	X	P(X)	XP(X)	0	$\frac{7}{10} \times \frac{7}{10} = \frac{49}{100}$	0	1	$\frac{3}{10} \times \frac{7}{10} \times 2 = \frac{42}{100}$	$\frac{42}{100}$	2	$\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$	$\frac{18}{100}$	<p>1/2</p> <p>1/2</p> <p>1 1/2</p> <p>1/2</p>
X	P(X)	XP(X)												
0	$\frac{7}{10} \times \frac{7}{10} = \frac{49}{100}$	0												
1	$\frac{3}{10} \times \frac{7}{10} \times 2 = \frac{42}{100}$	$\frac{42}{100}$												
2	$\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$	$\frac{18}{100}$												
30(b) Ans	<p>$A = \{(3,6), (4,5), (5,4), (6,3)\}$</p> <p>$P(A) = \frac{4}{36} = \frac{1}{9}$, $P(B) = \frac{30}{36} = \frac{5}{6}$</p> <p>$P(A \cap B) = \frac{3}{36} = \frac{1}{12}$</p> <p>$P(A) \times P(B) = \frac{5}{54} \neq P(A \cap B)$</p> <p>Therefore, A and B are not independent.</p> <p>A and B are not mutually exclusive as $A \cap B \neq \emptyset$</p>	<p>1</p> <p>1/2</p> <p>1</p> <p>1/2</p>												

31.	<p>f and g are continuous functions on interval [a, b]. Given that $f(a - x) = f(x)$</p> <p>and $g(x) + g(a - x) = a$, show that $\int_0^a f(x)g(x) dx = \frac{a}{2} \int_0^a f(x) dx$.</p>	
Ans	$I = \int_0^a f(x)g(x) dx$ $= \int_0^a f(a - x)g(a - x) dx$ $= \int_0^a f(x)[a - g(x)] dx$ $I = a \int_0^a f(x) dx - \int_0^a f(x)g(x) dx$ <p>Adding, we get $I = \frac{a}{2} \int_0^a f(x) dx$</p>	<p>1</p> <p>1</p> <p>1</p>
<p>SECTION-D</p> <p>This section comprises 4 Long Answer (LA) type questions of 5 marks each.</p>		
32	<p>(a) Find the shortest distance between the lines :</p> $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3} \text{ and}$ $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}.$ <p style="text-align: center;">OR</p> <p>(b) Find the image A' of the point A(2, 1, 2) in the line $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of line joining AA'. Find the foot of perpendicular from point A on the line l.</p>	
32(a) Ans	<p>The vector equations of the lines are</p> $\vec{r} = -\hat{i} + \hat{j} + 9\hat{k} + \lambda(2\hat{i} + \hat{j} - 3\hat{k})$ $\vec{r} = 3\hat{i} - 15\hat{j} + 9\hat{k} + \mu(2\hat{i} - 7\hat{j} + 5\hat{k})$ $\vec{a}_1 = -\hat{i} + \hat{j} + 9\hat{k}, \vec{a}_2 = 3\hat{i} - 15\hat{j} + 9\hat{k}$ $\vec{b}_1 = 2\hat{i} + \hat{j} - 3\hat{k}, \vec{b}_2 = 2\hat{i} - 7\hat{j} + 5\hat{k}$ $\vec{a}_2 - \vec{a}_1 = 4\hat{i} - 16\hat{j}$	<p>1</p> <p>1</p>

	$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 2 & -7 & 5 \end{vmatrix} = -16\hat{i} - 16\hat{j} - 16\hat{k}$ $\text{S.D.} = \frac{ (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) }{ \vec{b}_1 \times \vec{b}_2 } = \frac{12}{\sqrt{3}} = 4\sqrt{3}$ <p style="text-align: center;">OR</p>	<p style="text-align: center;">2</p> <p style="text-align: center;">1</p>
<p>32(b) Ans</p>	<div style="text-align: center;">  </div> <p>Let the image of A in the line be $A'(\alpha, \beta, \gamma)$</p> <p>The point P, which is the point of intersection of the lines l and AA', will have coordinates $(\lambda + 4, -\lambda + 2, -\lambda + 2)$ for some λ.</p> <p>Drs of AP are $\langle \lambda + 2, -\lambda + 1, -\lambda \rangle$</p> <p>$AP \perp l$</p> $(\lambda + 2) - (-\lambda + 1) - (-\lambda) = 0$ $\Rightarrow \lambda = -\frac{1}{3}$ <p>Therefore, the coordinates of P are $(\frac{11}{3}, \frac{7}{3}, \frac{7}{3})$</p> <p>P is the mid-point of AA'</p> $\Rightarrow \frac{2 + \alpha}{2} = \frac{11}{3}, \frac{1 + \beta}{2} = \frac{7}{3}, \frac{2 + \gamma}{2} = \frac{7}{3}$ $\Rightarrow \alpha = \frac{16}{3}, \beta = \frac{11}{3}, \gamma = \frac{8}{3}$ <p>The coordinates of the image are $(\frac{16}{3}, \frac{11}{3}, \frac{8}{3})$</p> <p>The equation of AA' is</p>	<p style="text-align: center;">$\frac{1}{2}$</p> <p style="text-align: center;">$\frac{1}{2}$</p> <p style="text-align: center;">1</p> <p style="text-align: center;">$\frac{1}{2}$</p> <p style="text-align: center;">$1\frac{1}{2}$</p>

	$\frac{x-2}{\frac{10}{3}} = \frac{y-1}{\frac{8}{3}} = \frac{z-2}{\frac{2}{3}}$ <p>or,</p> $\frac{3(x-2)}{5} = \frac{3(y-1)}{4} = \frac{3(z-2)}{1}$	1
33.	Find : $\int \frac{5x}{(x+1)(x^2+9)} dx.$	
Ans	$\frac{5x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$ $\Rightarrow A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{9}{2}$ <p>Given integral</p> $= -\frac{1}{2} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{x+9}{x^2+9} dx$ $= -\frac{1}{2} \int \frac{1}{x+1} dx + \frac{1}{4} \int \frac{2x}{x^2+9} dx + \frac{1}{4} \int \frac{18}{x^2+9} dx$ $= -\frac{1}{2} \log x+1 + \frac{1}{4} \log(x^2+9) + \frac{3}{2} \tan^{-1} \frac{x}{3} + C$	2 1½ 1½
34	<p>(a) Given $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB. Hence, solve the system of linear equations :</p> $x - y + z = 4$ $x - 2y - 2z = 9$ $2x + y + 3z = 1$ <p style="text-align: center;">OR</p> <p>(b) If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find A^{-1}.</p> <p>Hence, solve the system of linear equations :</p> $x - 2y = 10$ $2x - y - z = 8$ $-2y + z = 7$	

<p>34(a)</p> <p>Ans</p>	$AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$ <p>The system of equations is equivalent to the matrix equation:</p> $BX = C, \text{ where } C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\Rightarrow X = B^{-1}C$ $AB = 8I$ $\Rightarrow B^{-1} = \frac{1}{8}A$ $X = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$ $\therefore x = 3, y = -2, z = -1$ <p style="text-align: center;">OR</p>	<p>2</p> <p>½</p> <p>1</p> <p>1½</p>
<p>34(b)</p> <p>Ans</p>	$ A = 1 \neq 0 \Rightarrow A^{-1} \text{ exists.}$ $\text{adj}A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ $A^{-1} = \frac{1}{ A } \text{adj}A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ <p>The given system of equations is equivalent to the matrix equation</p> $A^T X = B, \text{ where } B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\Rightarrow X = (A^T)^{-1}B$ $\Rightarrow X = (A^{-1})^T B$ $\Rightarrow X = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$ $\therefore x = 0, y = -5, z = -3$	<p>1</p> <p>1½</p> <p>½</p> <p>½</p> <p>1½</p>

35.	Using integration, find the area of the region bounded by the line $y = 5x + 2$, the x - axis and the ordinates $x = -2$ and $x = 2$.	
Ans	 <p>The required area</p> $= \left \int_{-2}^{-\frac{2}{5}} (5x + 2) dx \right + \int_{-\frac{2}{5}}^2 (5x + 2) dx$ $= \left \left[\frac{(5x + 2)^2}{10} \right]_{-2}^{-\frac{2}{5}} \right + \left[\frac{(5x + 2)^2}{10} \right]_{-\frac{2}{5}}^2$ $= \frac{64}{10} + \frac{144}{10} = \frac{104}{5}$	<p>Correct sketch and shading</p> <p>2</p> <p>1</p> <p>1</p> <p>1</p>

	$P(A) = \frac{60}{100}, P(B) = \frac{30}{100}, P(C) = \frac{10}{100}$ $P(E) = P(A) \times P\left(\frac{E}{A}\right) + P(B) \times P\left(\frac{E}{B}\right) + P(C) \times P\left(\frac{E}{C}\right)$ $= \frac{60}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{90}{100} + \frac{10}{100} \times \frac{95}{100}$ $= \frac{845}{1000} \text{ or } \frac{169}{200}$	<p>1/2</p> <p>1</p> <p>1/2</p>
36(ii) Ans	$P\left(\frac{C}{E}\right) = \frac{P(C) \times P\left(\frac{E}{C}\right)}{P(E)}$ $= \frac{\frac{10}{100} \times \frac{5}{100}}{\frac{60}{100} \times \frac{20}{100} + \frac{30}{100} \times \frac{10}{100} + \frac{10}{100} \times \frac{5}{100}}$ $= \frac{50}{\frac{10000}{1550}} = \frac{1}{31}$	1
36(iii) Ans	$P\left(\frac{A \text{ or } B}{E}\right) = 1 - P\left(\frac{C}{E}\right) = 1 - \frac{1}{31} = \frac{30}{31}$	1
37.	 <p>A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres.</p> <p>Based on the above, answer the following :</p> <p>(i) Find the intervals on which the $f(x)$ is increasing or decreasing, $x \in [0, \pi]$. 2</p> <p>(ii) Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion. 2</p>	
(i) Ans	$f'(x) = e^x(\cos x + \sin x)$ <p>For critical points, $f'(x) = 0$</p> $\Rightarrow \cos x + \sin x = 0$ $\Rightarrow \cos x = -\sin x$	1/2

	<p>For x to be a critical point $x \in (0, \pi)$, hence, $x = \frac{3\pi}{4}$</p> <p>For all $x \in \left[0, \frac{3\pi}{4}\right], f'(x) \geq 0$</p> <p>Hence, f is increasing in $\left[0, \frac{3\pi}{4}\right]$</p> <p>Note: If a student concludes the answer in any of the following intervals, full marks may be awarded: $\left(0, \frac{3\pi}{4}\right)$ or $\left[0, \frac{3\pi}{4}\right)$ or $\left(0, \frac{3\pi}{4}\right]$</p> <p>For all $x \in \left[\frac{3\pi}{4}, \pi\right], f'(x) \leq 0$</p> <p>Hence, f is decreasing in $\left[\frac{3\pi}{4}, \pi\right]$</p> <p>Note: If a student concludes the answer in any of the following intervals, full marks may be awarded: $\left(\frac{3\pi}{4}, \pi\right)$ or $\left(\frac{3\pi}{4}, \pi\right]$ or $\left[\frac{3\pi}{4}, \pi\right)$</p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
37(ii) Ans	<p>$x = \frac{3\pi}{4}$ is a critical point</p> <p>$f''(x) = e^x(\cos x - \sin x) + e^x(\cos x + \sin x)$</p> <p>$= 2e^x \cos x$</p> <p>$f''\left(\frac{3\pi}{4}\right) = -ve$</p> <p>Hence, $\frac{3\pi}{4}$ is a point of local maximum.</p>	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>

38.	<p>A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}$.</p>  <p>Based on the above, answer the following :</p> <p>(i) How many relations can be there from S to J ? 1</p> <p>(ii) A student identifies a function from S to J as $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$ Check if it is bijective. 1</p> <p>(iii) (a) How many one-one functions can be there from set S to set J ? 2</p> <p style="text-align: center;">OR</p> <p>(iii) (b) Another student considers a relation $R_1 = \{(S_1, S_2), (S_2, S_4)\}$ in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric. 2</p>	
38 Ans (i)	The number of relations = $2^{4 \times 3} = 2^{12}$	1
38 Ans (ii)	Since, S_2 and S_3 have been assigned the same judge J_2 , the function is not one-one. Hence, it is not bijective.	1
38 (iii) (a)	There cannot exist any one-one function from S to J as $n(S) > n(J)$. Hence, the number of one-one functions from S to J is 0. OR	2
38 (iii) (b)	To make R_1 reflexive and not symmetric we need to add the following ordered pairs: $(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)$	2

