INDEFINITE INTEGRATION

1. If f & g are functions of x such that g′(x) = f(x) then,

$\int f(x)\,dx = g(x) + c \Leftrightarrow \frac{d}{dx}\{g(x)+c\} = f(x)$, where c is called the **constant of integration**.

2. **Standard Formula:**

(i) $\int (ax + b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c,\ n \neq -1$

(ii) $\int \frac{dx}{ax+b} = \frac{1}{a}\,\ell n\,(ax + b) + c$

(iii) $\int e^{ax+b}\,dx = \frac{1}{a}\,e^{ax+b} + c$

(iv) $\int a^{px+q}\,dx = \frac{1}{p}\,\frac{a^{px+q}}{\ell n\,a} + c;\ a > 0$

(v) $\int \sin(ax + b)\,dx = -\frac{1}{a}\cos(ax + b) + c$

(vi) $\int \cos(ax + b)\,dx = \frac{1}{a}\sin(ax + b) + c$

(vii) $\int \tan(ax + b)\,dx = \frac{1}{a}\,\ell n\,\sec(ax + b) + c$

(viii) $\int \cot(ax + b)\,dx = \frac{1}{a}\,\ell n\,\sin(ax + b) + c$

(ix) $\int \sec^2(ax + b)\,dx = \frac{1}{a}\tan(ax + b) + c$

(x) $\int \mathrm{cosec}^2(ax + b)\,dx = -\frac{1}{a}\cot(ax + b) + c$

(xi) $\int$ secx dx = ℓn (secx + tanx) + c **OR** ℓn tan $\left(\frac{\pi}{4} + \frac{x}{2}\right)$ + c

(xii) $\int$ cosec x dx = ℓn (cosecx – cotx) + c

OR ℓn tan $\frac{x}{2}$ + c **OR** – ℓn (cosecx + cotx) + c

(xiii) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$

(xiv) $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

(xv) $\int \frac{dx}{|x|\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a} + c$

(xvi) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \ell n\left[x + \sqrt{x^2 + a^2}\right] + c$

(xvii) $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ell n\left[x + \sqrt{x^2 - a^2}\right] + c$

(xviii) $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ell n\left|\frac{a+x}{a-x}\right| + c$

(xix) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ell n\left|\frac{x-a}{x+a}\right| + c$

(xx) $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

(xxi) $\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\ell n\left(\frac{x + \sqrt{x^2 + a^2}}{a}\right) + c$

(xxii) $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ell n\left(\frac{x + \sqrt{x^2 - a^2}}{a}\right) + c$

3. Integration by Subsitutions

If we subsitute f(x) = t, then f ′(x) dx = dt

4. Integration by Part :

$$\int \left(f(x)\, g(x)\right) dx = f(x) \int \left(g(x)\right) dx - \int \left(\frac{d}{dx}\left(f(x)\right) \int \left(g(x)\right) dx \right) dx$$

5. Integration of type $\int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \sqrt{ax^2+bx+c}\ dx$

Make the substitution $x + \frac{b}{2a} = t$

6. Integration of type

$$\int \frac{px+q}{ax^2+bx+c} dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}} dx, \int (px+q)\sqrt{ax^2+bx+c}\ dx$$

Make the substitution $x + \frac{b}{2a} = t$, then split the integral as some of two integrals one containing the linear term and the other containing constant term.

7. Integration of trigonometric functions

(i) $\int \frac{dx}{a+b\sin^2 x}$ **OR** $\int \frac{dx}{a+b\cos^2 x}$

OR $\int \frac{dx}{a\sin^2 x + b\sin x \cos x + c\cos^2 x}$ **put tan x = t.**

(ii) $\int \frac{dx}{a+b\sin x}$ **OR** $\int \frac{dx}{a+b\cos x}$

OR $\int \frac{dx}{a+b\sin x + c\cos x}$ **put** $\tan\frac{x}{2} = t$

(iii) $\int \frac{a.\cos x + b.\sin x + c}{\ell.\cos x + m.\sin x + n}$ **dx. Express Nr ≡ A(Dr) + B** $\frac{d}{dx}$ **(Dr) + c & proceed.**

8. **Integration of type**

$$\int \frac{x^2+1}{x^4+Kx^2+1}\,dx \quad \text{where K is any constant.}$$

Divide Nr & Dr by x^2 & put $x \mp \frac{1}{x}$ = t.

9. **Integration of type**

$$\int \frac{dx}{(ax+b)\sqrt{px+q}} \quad \textbf{OR} \quad \int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}} \text{ ; put } px + q = t^2.$$

10. **Integration of type**

$$\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}} \text{ , put } ax + b = \frac{1}{t}\text{ ;}$$

$$\int \frac{dx}{(ax^2+b)\sqrt{px^2+q}} \text{ , put } x = \frac{1}{t}$$

DEFINITE INTEGRATION

Properties of definite integral

1. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

2. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

3. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

4. $\int_{-a}^a f(x)\,dx = \int_0^a (f(x)+f(-x))\,dx = \begin{cases} 2\int_0^a f(x)dx & , \quad f(-x) = f(x) \\ 0 & , \quad f(-x) = -f(x) \end{cases}$

5. $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

6. $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

7. $\int_0^{2a} f(x)\, dx = \int_0^a (f(x) + f(2a-x))\, dx = \begin{cases} 2\int_0^a f(x)dx & , \quad f(2a-x) = f(x) \\ 0 & , \quad f(2a-x) = -f(x) \end{cases}$

8. If f(x) is a periodic function with period T, then

$\int_0^{nT} f(x)\, dx = n \int_0^T f(x) dx,\ n \in z, \quad \int_a^{a+nT} f(x)\, dx = n \int_0^T f(x) dx,\ n \in z,\ a \in R$

$\int_{mT}^{nT} f(x)\, dx = (n-m) \int_0^T f(x) dx,\ m, n \in z, \quad \int_{nT}^{a+nT} f(x)\, dx = \int_0^a f(x) dx,\ n \in z,\ a \in R$

$\int_{a+nT}^{b+nT} f(x)\, dx = \int_a^a f(x) dx,\ n \in z,\ a, b \in R$

9. If $\psi(x) \le f(x) \le \phi(x)$ for $a \le x \le b$,

then $\int_a^b \psi(x)\, dx \le \int_a^b f(x)\, dx \le \int_a^b \phi(x)\, dx$

10. If $m \le f(x) \le M$ for $a \le x \le b$, then $m\,(b-a) \le \int_a^b f(x) dx \le M\,(b-a)$

11. If $f(x) \ge 0$ on [a, b] then $\int_a^b f(x)\, dx \ge 0$

Leibnitz Theorem : If $F(x) = \int_{g(x)}^{h(x)} f(t)\, dt$, then $\frac{dF(x)}{dx} = h'(x)\, f(h(x)) - g'(x)\, f(g(x))$