

CAPACITANCE

1. (i) $q \propto V \Rightarrow q = CV$
 q : Charge on positive plate of the capacitor
 C : Capacitance of capacitor.
 V : Potential difference between positive and negative plates.

(ii) Representation of capacitor : $—| |—, —| (—$

(iii) Energy stored in the capacitor : $U = \frac{1}{2} CV^2 = \frac{Q^2}{2C} = \frac{QV}{2}$

(iv) Energy density = $\frac{1}{2} \epsilon_0 \epsilon_r E^2 = \frac{1}{2} \epsilon_0 K E^2$

ϵ_r = Relative permittivity of the medium.

$K = \epsilon_r$: Dielectric Constant

For vacuum, energy density = $\frac{1}{2} \epsilon_0 E^2$

(v) Types of Capacitors :

(a) **Parallel plate capacitor**

$$C = \frac{\epsilon_0 \epsilon_r A}{d} = K \frac{\epsilon_0 A}{d}$$

A : Area of plates

d : distance between the plates(\ll size of plate)

(b) **Spherical Capacitor :**

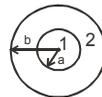
- Capacitance of an isolated spherical Conductor (hollow or solid)

$$C = 4 \pi \epsilon_0 \epsilon_r R$$

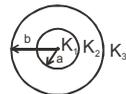
R = Radius of the spherical conductor

- Capacitance of spherical capacitor

$$C = 4\pi\epsilon_0 \frac{ab}{(b-a)}$$

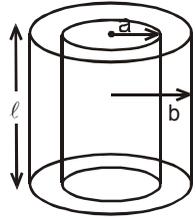


- $C = \frac{4\pi\epsilon_0 K_2 ab}{(b-a)}$



(c) **Cylindrical Capacitor :** $l \gg \{a, b\}$

$$\text{Capacitance per unit length} = \frac{2\pi\epsilon_0}{\ln(b/a)} \text{ F/m}$$



- (vi) Capacitance of capacitor depends on
 (a) Area of plates
 (b) Distance between the plates
 (c) Dielectric medium between the plates.
- (vii) Electric field intensity between the plates of capacitor

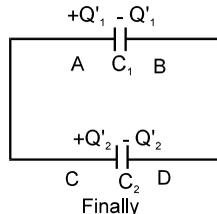
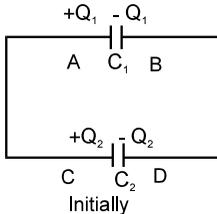
$$E = \frac{\sigma}{\epsilon_0} = \frac{V}{d}$$

σ : Surface charge density

- (viii) Force experienced by any plate of capacitor : $F = \frac{q^2}{2A\epsilon_0}$

2. DISTRIBUTION OF CHARGES ON CONNECTING TWO CHARGED CAPACITORS:

When two capacitors are C_1 and C_2 are connected as shown in figure



- (a) Common potential :

$$\Rightarrow V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{\text{Total charge}}{\text{Total capacitance}}$$

$$(b) \quad Q_1' = C_1 V = \frac{C_1}{C_1 + C_2} (Q_1 + Q_2)$$

$$Q_2' = C_2 V = \frac{C_2}{C_1 + C_2} (Q_1 + Q_2)$$

(c) Heat loss during redistribution :

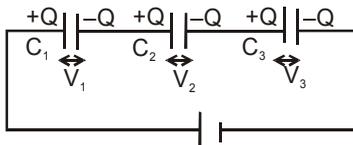
$$\Delta H = U_i - U_f = \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_1 - V_2)^2$$

The loss of energy is in the form of Joule heating in the wire.

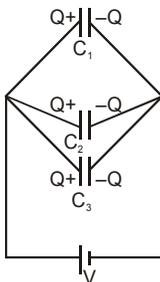
3. Combination of capacitor :

(i) Series Combination

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$



(ii) Parallel Combination :



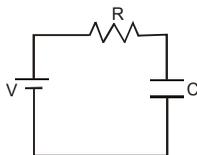
$$C_{\text{eq}} = C_1 + C_2 + C_3$$

$$Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$$

4. Charging and Discharging of a capacitor :

(i) Charging of Capacitor (Capacitor initially uncharged):

$$q = q_0 (1 - e^{-t/\tau})$$

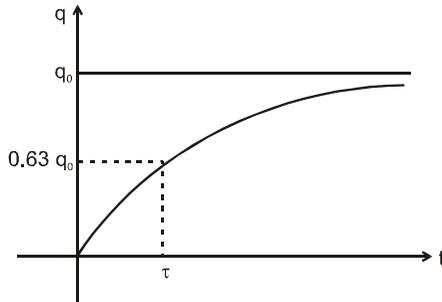


q_0 = Charge on the capacitor at steady state

$$q_0 = CV$$

τ : Time constant = CR_{eq} .

$$I = \frac{q_0}{\tau} e^{-t/\tau} = \frac{V}{R} e^{-t/\tau}$$

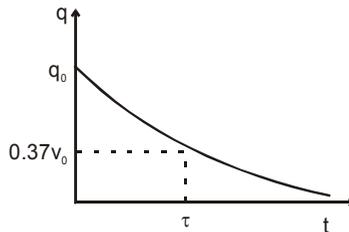
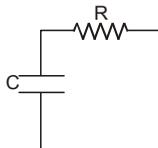


(ii) Discharging of Capacitor :

$$q = q_0 e^{-t/\tau}$$

q_0 = Initial charge on the capacitor

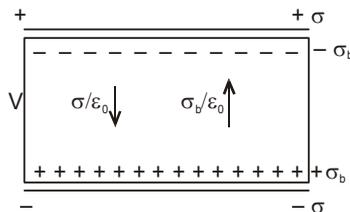
$$I = \frac{q_0}{\tau} e^{-t/\tau}$$



5. Capacitor with dielectric :

(i) Capacitance in the presence of dielectric :

$$C = \frac{K\epsilon_0 A}{d} = KC_0$$



C_0 = Capacitance in the absence of dielectric.

$$(ii) \quad E_{in} = E - E_{ind} = \frac{\sigma}{\epsilon_0} - \frac{\sigma_b}{\epsilon_0} = \frac{\sigma}{K\epsilon_0} = \frac{V}{d}$$

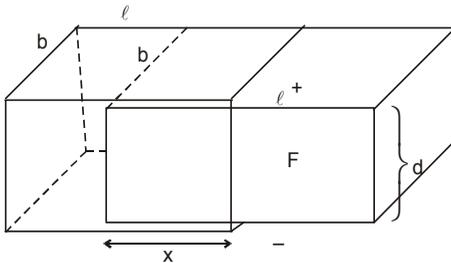
$E : \frac{\sigma}{\epsilon_0}$ Electric field in the absence of dielectric

$E_{ind} :$ Induced (bound) charge density.

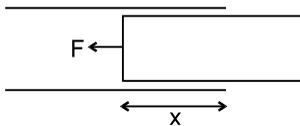
$$(iii) \quad \sigma_b = \sigma \left(1 - \frac{1}{K}\right).$$

6. Force on dielectric

(i) When battery is connected $F = \frac{\epsilon_0 b(K-1)V^2}{2d}$



(ii) When battery is not connected $F = \frac{Q^2}{2C^2} \frac{dC}{dx}$



* Force on the dielectric will be zero when the dielectric is fully inside.